

D 93426

(Pages : 4)

Name.....

Reg. No.....

**FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY)
EXAMINATION, NOVEMBER 2020**

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

Part A

*Answer all questions in Part A.
Each question carries a weightage of 1.*

1. State a necessary and sufficient condition for two simple graphs G and H to be isomorphic.
2. Prove or disprove : No loop can belong to an edge cut.
3. Is it true that every eulerian graph is connected. Justify.
4. If G is a connected plane graph, show that a cut edge of G belong to exactly one face.
5. Define maximal element and maximum element. Illustrate with an example.
6. If $(X, +, \cdot, ')$ is a Boolean algebra, prove that : (i) $x + x = x$ and (ii) $x \cdot x = x$ for $x \in X$.
7. Prove that $(uv)^R = v^R u^R$ for all $u, v \in \Sigma^+$.
8. Define a finite automaton. Illustrate with an example.

(8 × 1 = 8 weightage)

Part B

*Answer six questions in Part B choosing two from each unit.
Each question carries a weightage of 2.*

Unit I

9. (a) Show that the number of edges of a simple graph with n vertices and w components is :

$$\frac{(n-w)(n-w+1)}{2}$$

- (b) Is it true ; if G is a simple graph with $\delta \geq \frac{n-2}{2}$, then G is connected. Justify.

Turn over

10. (a) Prove that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.
- (b) Show that the number of edges of a tree on n vertices is $n - 1$.
11. (a) If G is a plane graph and f is a face of G then prove that there exists a plane embedding of G in which f is the exterior face.
- (b) Prove or disprove : All planar embeddings of a given planar graph have the same number of faces.

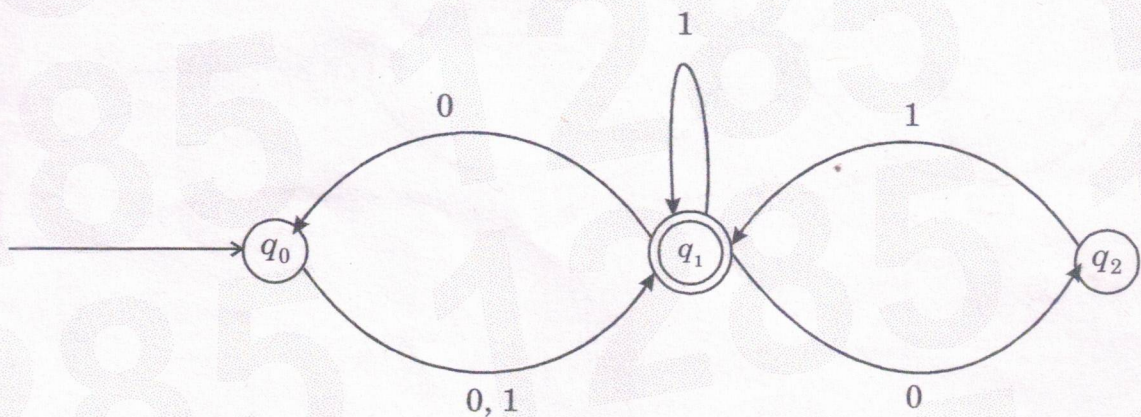
Unit II

12. (a) Let X be a set and \leq be a binary relation on X which is reflexive and transitive. Define R on X by $x R y$ if $x \leq y$ and $y \leq x$. Verify whether R is an equivalence relation.
- (b) Prove or disprove : A longest chain in a partially ordered set (X, \leq) is maximal, but the converse need not hold.
13. If $(X, +, \cdot, ')$ is a Boolean algebra, prove that, for all $x, y, z \in X$.
- (a) $x + (y + z) = (x + y) + z$ and (b) $(x')' = x$.
14. (a) What is the disjunctive normal form of a Boolean function.
- (b) Write the Boolean function $f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$ in its disjunctive normal form.

Unit III

15. (a) Find a *dfa* that accepts the set of all strings with exactly one a where $\Sigma = \{a, b\}$.
- (b) Find an *nfa* that accepts $\{a\}^*$ and is such that if in its transition graph a single edge is removed without any other changes, the resulting automaton accepts $\{a\}$.
16. Find a *dfa* that accepts the set of all strings on $\Sigma = \{a, b\}$ starting with the Prefix ab .

17. Which of the given strings are accepted by the following automaton : (i) 01001 ; (ii) 10010 ; (iii) 000.



(6 × 2 = 12 weightage)

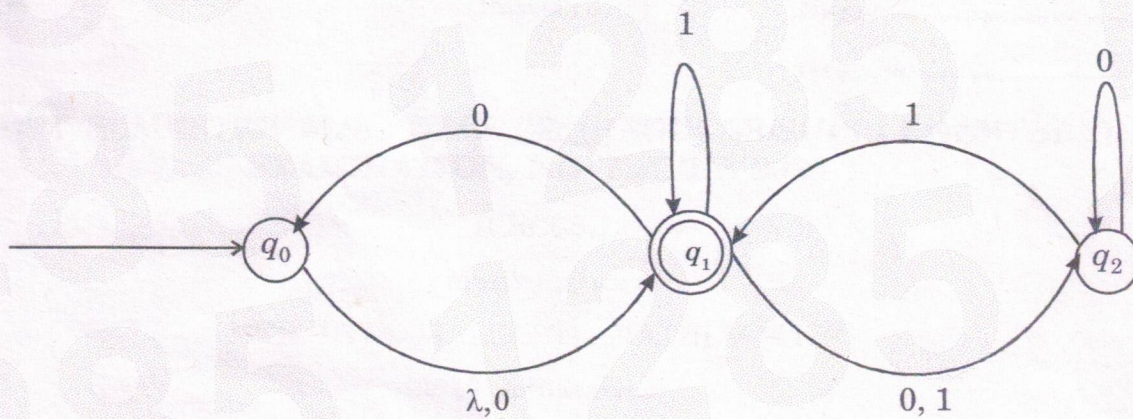
Part C

Answer any **two** questions in Part C.

Each question carries a weightage of 5.

18. (a) Let T be a graph with n vertices, then prove the following are equivalent :
- T is a tree.
 - T has no cycles and has $n - 1$ edges.
 - There exists a unique path between any two vertices in T .
- (b) Find all non-isomorphic trees with 5 vertices.
19. (a) Prove that K_5 is non-planar.
- (b) For any simple planar graph G , prove that $\delta(G) \leq 5$.
20. (a) Show that every Boolean function on n variables x_1, x_2, \dots, x_n can be uniquely expressed as the sum of terms of the form $x_1^{\epsilon_1} x_2^{\epsilon_2} \dots x_n^{\epsilon_n}$ where each $x_i^{\epsilon_i}$ is either x_i or x_i' .
- (b) Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

21. Convert the *nfa* given by the following diagram into an equivalent *dfa*.



(2 × 5 = 10 weightage)