C 23879 **(Pages : 3)** **Name**.....

Reg. No.....

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MAT 2C 02—MATHEMATICS–2

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum Marks 20.

1. State inverse function test. Verify that $f(x) = x^2 + x$ has an inverse if f is defined on $\left[\frac{-1}{2}, \infty\right)$.

2. Find the slope of the line tangent to the graph of $r = 3\cos^2 2\theta$ at $\theta = \frac{\pi}{6}$.

3. Compute $\int \frac{\sinh x dx}{1 + \cosh^2 x}$.

4. Prove that $\tanh^{-1} x = \frac{1}{2} \ln\left[\frac{1+x}{1-x}\right], -1 < x < 1$.

5. For which values of the exponent r is $\int_1^\infty x^r \, dx$ convergent ?

6. State Simpson's rule.

7. Sum the series $\sum_{i=0}^{\infty} \frac{3^i - 2^i}{6^i}$.

Turn over

8. State alternating series test and test the convergence for the series $\sum_{i=1}^{\infty} \frac{(-i)^i}{(1+i)^2}$.

9. Prove that the vectors $w_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), w_2 = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $w_3 = \left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are orthonormal vectors.

10. Determine whether the set of all functions f with $f(1) = 0$ is a subspace of $C(-\infty, \infty)$.

11. Find the inverse of $A = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}$.

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} -1 & 2 \\ -7 & 8 \end{pmatrix}$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum Marks 30.

13. Find the length of the graph of $f(x) = (x-1)^{\frac{3}{2}} + 2$ on $[0, 2]$.

14. Find the area of the surface obtained by revolving the graph of x^3 on $[0, 1]$.

15. State ratio test for the power series. For which x does $\sum_{n=0}^{\infty} \frac{i}{i+1} x^i$ converge.

16. Find the Maclaurin series for $f(x) = \sin x$.

17. Use Grami-Schmidt orthonormalization process to transform the basis $\{u_1, u_2, u_3\}$ for $\mathbb{R}^3$ into an orthonormal basis $B' = \{w_1, w_2, w_3\}$, where $u_1 = (1, 1, 1), u_2 = (1, 2, 2)$ and $u_3 = (1, 1, 0)$.

18. State Cayley's theorem and using this theorem find the inverse of $A = \begin{pmatrix} -2 & 4 \\ -1 & 3 \end{pmatrix}$.

19. Diagonalize $A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{pmatrix}$.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum Marks 10.

20. (a) Find the area enclosed by the cardiod $r = 1 + \cos\theta$.

(b) Using LU factorization to solve the system linear equations $AX = B$ where $A = \begin{pmatrix} 2 & -2 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$.

21. (a) Calculate $\sin\left(\frac{\pi}{4} + 0.06\right)$ to whithin 0.0001 by using Taylor series about $x_0 = \frac{\pi}{4}$.

(b) Determine whether the vectors $u_1 = (1, -1, 3, -1)$, $u_2 = (1, -1, 4, 2)$ and $u_3 = (1, -1, 5, 7)$ are linearly dependent or independent.

(1 × 10 = 10 marks)