C 4750 **(Pages : 3)** **Name....................................**

Reg. No...............................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2021

(CBCSS)

Mathematics

MT 2C 08—TOPOLOGY

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *the questions.*
Each question has weightage 1.

1. Define co-finite topology and co-countable topology on a set. Among the two which is stronger topology ? Justify your answer.
2. Define base for a topology. Give an example for a base for the usual topology on the set of real numbers.
3. Prove that in a topological space, the closure of the closure of a set is same as the closure of the set.
4. When do we say that a topological property is divisible ? Prove that the property of being a finite space is divisible.
5. Prove that every separable space satisfies the countable chain condition.
6. Prove that the topological product of any finite number of connected spaces is connected.
7. Write an example of a Hausdorff topology on the set $X = \{1, 2, 3\}$.
8. State Urysohn's lemma.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **two** *questions from each unit.*
Each question has weightage 2.

Unit I

9. Prove that intersection of two open sets in a metric space is open.

10. Determine the topology induced by a discrete metric on a set.

11. If a space is second countable, then prove that every open cover of it has a countable subcover.

Unit II

12. Prove that every closed surjective map is a quotient map.

13. Let X be a compact space and suppose $f : X \to Y$ is continuous and onto. Then prove that Y is compact.

14. Prove that every second countable space is first countable.

Unit III

15. Suppose y is an accumulation point of a subset A of a T_1 space X. Then prove that every neighbourhood of y contains infinitely many points of A.

16. Prove that a compact subset of a Hausdorff space is closed.

17. Let A, B be subsets of a space X and suppose there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$. Then prove that there exist disjoint open sets U, V such that $A \subset U$ and $B \subset V$.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Prove that the usual topology on the Euclidean plane R^2 is strictly weaker than the topology induced by lexicographic ordering.

    (b) Determine the topology induced by a discrete metric on a set.

19. (a) Prove that if the space (X, T) has a base $\mathbb{B}$ of cardinality α, then the cardinality of T cannot exceed 2^{α}.

    (b) Let (X, T) be a topological space and $\mathbb{B} \subset T$. Then prove that $\mathbb{B}$ is a base for T if and only if for any $x \in X$ and any open set G containing x, there exists $B \in \mathbb{B}$ such that $x \in B$ and $B \subset G$.

20. (a) Let (X, d) be a compact metric space and U be an open cover of X. Then prove that there exists a positive real number r such that for any $x \in X$, there exits $V \in U$ such that $B(x, r) \subset V$.

    (b) Let $f : X \to Y$ be a continuous surjection. Then if X is connected, prove that Y is also connected.

21. A be a closed subset of a normal space X and suppose $f : A \to [-1, 1]$ is a continuous function. Then prove that there exists a continuous function $F : X \to [-1, 1]$ such that $F(x) = f(x)$ for all $x \in A$.

(2 × 5 = 10 weightage)