C 80712 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCSS)

Mathematics

MT 4E 11—GRAPH THEORY

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** *questions.*
Each question carries weightage 1.

1. Prove that every non-trivial tree has at least two vertices of degree one.

2. In a tree, prove that every two vertices are connected by a unique path.

3. If G is connected, prove that $\epsilon \geq \upsilon - 1$.

4. If G is 2-connected prove that any two vertices of G lie on a common cycle.

5. If G is a block with $\upsilon(G) \geq 3$, prove that any two edges of G lie on a common cycle.

6. If G is Hamiltonian, prove that $w(G - S) \leq |S|$ for every non-empty proper subset S.

7. Let $S \subseteq V$. If $V \setminus S$ is a covering of G, prove that S is an independent set of G.

8. With usual notations, prove that $\alpha + \beta = \delta$.

9. Prove that no vertex cut is a clique in a critical graph.

10. Prove that a tree G has a perfect matching if $o(G - \upsilon) = 1$ for all $\upsilon \in V(G)$.

11. Prove that every subgraph of a planar graph is planar.

12. Find $\chi'(K_5)$ by exhibiting appropriate edge colouring.

Turn over

13. Prove with necessary details that every k-chromatic graph has at least k vertices of degree at least $k-1$.

14. Prove that all planar embedding of a connected planar graph have the same number of faces.

(14 × 1 = 14 weightage)

Part B (Paragraph Type)

Answer any **seven** *questions from the following ten questions.*
Each question carries weightage 2.

15. If G is a graph with $\upsilon - 1$ edges, prove that the following are equivalent :

    (a) G is connected.

    (b) G is a tree.

16. Prove that a non-empty connected graph is Eulerian if and only if it has no vertices of odd degree.

17. Prove that an edge e of G is a cut edge if and only if e is contained in no cycle of G.

18. If e is a link of G, prove that $\tau(G) = \tau(G - e) + \tau(G \cdot e)$.

19. Define closure c (G) of a graph G. Give a non-trivial example.

20. If G is Eulerian, prove that any trail in G constructed by Fleury's algorithm is an Eulerian tour of G.

21. Prove that every 3-regular graph without cut edges has a perfect matching.

22. Prove that $r(k, k) \geq 2^{\frac{k}{2}}$.

23. Let G be k-critical graph with 2-vertex cut $\{u, v\}$. Prove that $d(u) + d(v) \geq 3k - 5$.

24. Let v be a vertex of a planar graph G. Prove that G can be embedded in the plane in such a way that v is on the exterior face of the embedding.

(7 × 2 = 14 weightage)

Part C (Essay Type)

Answer any **two** *questions from the following four questions.*
Each question carries weightage 4.

25. For $m < n$, let $f(m, n)$ be the least number of edges that an m-connected graph on n vertices can have. Construct an m-connected graph on n vertices with $f(m, n) = \left\{\frac{mn}{2}\right\}$.

26. Prove the following :

    (i) If G is a non-Hamiltonian simple graph with $v > 3$, then G is degree majorised by some $C_{m,v}$.

    (ii) Any Hamiltonian graph is 2-connected.

27. Prove that each vertex of a diconnected tournament D with $v \geq 3$ is contained in a directed k-cycle where $3 \leq k \leq v$.

28. State and prove Kuratowski's theorem.

(2 × 4 = 8 weightage)