

C 20645

(Pages : 3)

Name.....

Reg. No.....

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS-UG)

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

1. Define continuity of a function. Show that the constant function $f(x) = b$ is continuous on \mathbb{R} .
2. State Boundedness theorem. Is boundedness of the interval, a necessary condition in the theorem? Justify with an example.
3. If $f : A \rightarrow \mathbb{R}$ is uniformly continuous on $A \subseteq \mathbb{R}$ and (x_n) is a Cauchy sequence in A . Then show that $f(x_n)$ is a Cauchy sequence in \mathbb{R} .
4. Define Riemann sum of a function $f : [a, b] \rightarrow \mathbb{R}$.
5. Suppose f and g are in $\mathbb{R}[a, b]$ then show that $f + g$ is in $\mathbb{R}[a, b]$.
6. State squeeze theorem for Riemann integrable functions.
7. If f belong to $\mathbb{R}[a, b]$, then show that its absolute value $|f|$ is in $\mathbb{R}[a, b]$.
8. Define pointwise convergence of a sequence (f_n) of functions.
9. Discuss the uniform convergence of $f_n(x) = x^n$ on $(-1, 1]$.
10. If $h_n(x) = 2nxe^{-nx^2}$ for $x \in [0, 1], n \in \mathbb{N}$ and $h(x) = 0$ for all $x \in [0, 1]$, then show that :

$$\lim_{n \rightarrow \infty} \int_0^1 h_n(x) dx \neq \int_0^1 h(x) dx.$$

11. State Cauchy criteria for uniform convergence series of functions.

Turn over

12. Evaluate $\int_{-1}^0 \frac{dx}{\sqrt[3]{x}}$.

13. What is Cauchy principle value. Find the principal value of $\int_{-1}^1 \frac{dx}{x}$.

14. State Leibniz rule for differentiation of Ramann integrals.

15. State that $\Gamma(p+1) = p\Gamma(p)$ for $p > 0$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** questions.
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 30.

16. Show that the Dirichlet's function :

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases} \text{ is not continuous at any point of } \mathbb{R}.$$

17. State and prove Bolzano intermediate value theorem.

18. Show that the following functions are not uniformly continuous on the given sets :

(a) $f(x) = x^2$ on $A = [0, \infty]$.

(b) $g(x) = \sin \frac{1}{x}$ on $B = (0, \infty)$.

19. If $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$, then show that $f \in \mathbb{R}[a, b]$.

20. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \rightarrow \mathbb{R}$. Then show that f is continuous on A .

21. Let $f_n : [0, 1] \rightarrow \mathbb{R}$ be defined for $n \geq 2$ by :

$$f_n(x) = \begin{cases} n^2x & , 0 \leq x \leq \frac{1}{n} \\ -n^2(x - 2/n), & \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0 & , \frac{2}{n} \leq x \leq 1. \end{cases}$$

Show that the limit function is Riemann integrable. Verify whether $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx$.

22. Given $\iint_{\mathbb{R}^2} e^{-(x^2+y^2)} dx dy = \pi$, find the value of $\int_0^{\infty} e^{-x^2} dx = \sqrt{\frac{\pi}{2}}$.

23. Show that $\forall p > 0, q > 0$ $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

(5 × 6 = 30 marks)

Section C*Answer any two questions.**Each question carries 10 marks.*

24. State and prove Location of roots theorem.

25. State and prove Additivity theorem.

26. Evaluate (a) $\lim_{x \rightarrow \infty} \frac{x^n}{1+x^n}$ for $x \in \mathbb{R}, x \geq 0$. (b) $\lim_{x \rightarrow \infty} \frac{\sin nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.

Discuss about their uniform convergence.

27. (a) Show that $\forall q > -1, \int_0^1 x^q e^{-x} dx$ converges.

(b) Show that $\forall q \leq -1, \int_0^1 x^q e^{-x} dx$ diverges.

(2 × 10 = 20 marks)