**C 60053** (Pages : 4) Name.................................

Reg. No.................................

# SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, MARCH 2019

(CUCBCSS)

Mathematics

MAT 6B 13 (E 02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

## Section A

*Answer all the* **twelve** *questions.*
*Each question carries* 1 *mark.*

1. Define a convex set.
2. What is a degenerate solution of an L.P.P. ?
3. What is a slack variable ?
4. Write the names of any two methods to a solve a transportation problem.
5. Write the following L.P.P. in standard form :

   Maximize $Z = 2x_1 - 8x_2$

   subject to $x_1 + x_2 \geq -1$

   $x_1 - x_2 \leq 3$

   $x_1, x_2 \geq 0.$
6. Show that $x_1 = 2, x_2 = 1$ is a feasible solution of the L.P.P. given below :

   Maximize $Z = 4x_1 + x_2$

   $x_1 + x_2 \leq 3$

   $x_1 - x_2 > 1$

   $x_1, x_2 \geq 0.$
7. Explain, why we not use 'Transportation Algorithm' to solve 'An assignment problem'.
8. Find the number of possible feasible solutions of the following L.P.P. :

   Maximize $Z = x_1 + x_2$

   subject to the constraints $x_1 + x_2 + x_3 \leq 5$

   $x_1, x_2, x_3 \geq 0.$
9. Write the necessary and sufficient condition for a basic feasible solution to a L.P.P. to be an optimum (maximum).
10. Write the following L.P.P. in matrix form :

    Minimize $Z = x_1 + x_2 - x_3$

    subject to $x_1 + x_3 \geq 2$

    $x_1 - x_2 \geq 4$

    $x_1, x_2 \geq 0.$

Turn over

11. Write the dual of the following L.P.P. :

Minimize $Z = x_1 + x_2$

subject to $x_1 + x_2 \geq 1$

$x_1 - x_2 \leq 1$

$x_1, x_2 \geq 0.$

12. When we say that a 'transportation problem' is unbalanced ?

(12 × 1 = 12 marks)

## Section B

*Answer any* **nine** *out of twelve questions.*
*Each question carries 2 marks.*

13. Define a hyper sphere in $\mathbb{R}^n$.

14. Show that the following L.P.P. has no solution :

Maximize $Z = x_1 + x_2$

where $x_1 - x_2 \geq 0$

$3x_1 - x_2 \leq -3$

$x_1, x_2 \geq 0.$

15. Show that the intersection of two convex set is also a convex set.

16. Write a short note on 'The North-West Corner Rule'.

17. Write a short note on 'The Assignment Problem'.

18. Form the Mathematical formulation of the problem given below :

Prabha goes to the market to purchase buttons. She needs atleast 20 large buttons and 30 small buttons respectively. The shopkeeper sells buttons in two tons—(i) boxes ; and (ii) cards. A box contains 10 large buttons and 5 small buttons respectively ; whereas a card contains 2 large buttons and 5 small buttons respectively.

Determine the most economical way in which Prabha should purchase the buttons, if a box costs Rs. 25 and a card costs Rs. 10 only.

19. Write the dual problem of the following L.P.P. :

Maximize $f(x) = 2x_1 + 5x_2 + 6x_3$

subject to the constraints $5x_1 + 6x_2 - x_3 \leq 3$

$-2x_1 + x_2 + 4x_3 \leq 4$

$x_1 - 5x_2 + 3x_3 \leq 1$

$-3x_1 - 3x_2 + 7x_3 \leq 6$

$x_1, x_2, x_3 \geq 0.$

20. Find any basic feasible solution of the following transportation problem :

| | $m_1$ | $m_2$ | $m_3$ | $m_4$ | |
|---|---|---|---|---|---|
| $w_1$ | 1 | 2 | 3 | 4 | 2 |
| $w_2$ | 4 | 3 | 2 | 1 | 2 |
| $w_3$ | 2 | 1 | 4 | 3 | 3 |
| | 3 | 2 | 1 | 1 | |

21. Given that $a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, c_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ can be written in the form $a_1 = \lambda_1 b_1 + \lambda_2 c_1$. Find $\lambda_1$ and $\lambda_2$.

22. Write the Mathematical formulation of an assignment problem.

23. Check whether the set $A = \{(x_1, x_2) / x_1, x_2 \in \mathbb{R}\ s\ x_1^2 + x_2^2 = 1\}$ is a convex set.

24. Show that the vectors $a_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, b_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, c_1 = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ are linearly dependent.

(9 × 2 = 18 marks)

## Section C

*Answer any **six** out of nine questions.*
*Each question carries 5 marks.*

25. Maximize $Z = 2x_1 + x_2$

where $x_1 + x_2 \leq 4$

$x_1 + 2x_2 \leq 6$

$x_1 \leq 3$

$x_1, x_2 \geq 0.$

26. Prove that a hyperplane is a convex set.

27. Prove that the set of all feasible solutions to a L.P.P. constitutes a convex set.

28. Use simplex method to solve the following L.P.P. :

Maximize $Z = 2.5x_1 + x_2$

subject to the constraints $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0.$

29. Find a basic feasible solution of the following transportation problem by using Vogel's approximation method :

| | I | II | III | IV | |
|---|---|---|---|---|---|
| A | 5 | 1 | 3 | 3 | 34 |
| B | 3 | 3 | 5 | 4 | 15 |
| C | 6 | 4 | 4 | 3 | 12 |
| D | 4 | –1 | 4 | 2 | 19 |
| | 21 | 25 | 17 | 17 | 80 |

30. Consider the problem of assigning five jobs to five persons. The assignment cost are given below :

| | | Job 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Person | A | 8 | 4 | 2 | 6 | 1 |
| | B | 0 | 9 | 5 | 5 | 4 |
| | C | 3 | 8 | 9 | 2 | 6 |
| | D | 4 | 3 | 1 | 0 | 3 |
| | E | 9 | 5 | 8 | 9 | 5 |

Determine the optimum assignment schedule.

31. Show that the following L.P.P. has an unbounded solution :

Maximize $Z = 4x_1 + 5x_2$

subject to $x_1 + x_2 \geq 1$

$-2x_1 + x_2 \leq 1$

$4x_1 - 2x_2 \leq 1$

$x_1, x_2 \geq 0.$

32. The column vector [1, 1, 1] is a feasible solution to the system of equations :

$x_1 + x_2 + 2x_3 = 4$

$2x_1 - x_2 + x_3 = 2.$

Reduce the given solution to a basic feasible solution.

33. State and prove 'Minimax Theorem'.

(6 × 5 = 30 marks)

## Section D

*Answer any* **two** *out of three questions.*
*Each question carries* 10 *marks.*

34. S.T. any convex combination of K different optimum solutions to a L.P.P. is again an optimum solution to the problem.

35. Use simplex method to solve :

Maximize $Z = 107x_1 + x_2 + 2x_3$

subject to the constraints $14x_1 + x_2 - x_3 + 3x_4 = 7$

$16x_1 + \frac{1}{2}x_2 - 2x_3 \leq 3$

$3x_1 \geq 0$

$x_1, x_2, x_3, x_4 \geq 0.$

36. Prove that a hyperplane is a closed set.

(2 × 10 = 20 marks)