C 80274 (Pages : 4) Name..................................

Reg. No..............................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 13 (E02)—LINEAR PROGRAMMING

Time : Three Hours Maximum : 80 Marks

Section A

Answer **all** *the twelve questions.*
Each question carries 1 mark.

1. Define a convex set.
2. Determine the convex hull of the set $A = \left\{(x_1, x_2) : x_1^2 + x_2^2 = 1\right\}$.
3. State graphical solution algorithm for an LPP involving two variables.
4. Define slack and surplus variables.
5. State the condition of optimality for a basic feasible solution to an LPP to be maximum.
6. Define artificial variable.
7. Write down the following LPP in standard form :

   Maximize $Z = 3x_1 + 2x_2 + 5x_3$

   subject to the constraints : $2x_1 - 3x_2 \leq 3$

   $$x_1 + 2x_2 + 3x_3 \geq 5$$

   $$3x_1 + 2x_3 \leq 2$$

   $$x_1, x_2 \geq 0$$

   and $x_3 \geq 0$.
8. Write the dual of the following LPP :

   Maximize $Z = 3x_1 - x_2 + x_3$

   subject to the constraints : $4x_1 - x_2 \leq 8$

   $$8x_1 + x_2 + 3x_3 \geq 12$$

   $$5x_1 - 6x_3 \leq 13$$

   $$x_1, x_2, x_3 \geq 0.$$

Turn over

9. State fundamental theorem of linear programing.
10. Define triangular basis in a transportation problem.
11. What is an assignment problem ?
12. What is degeneracy in transportation problem ?

(12 × 1 = 12 marks)

Section B

Answer any **nine** *out of twelve questions.*
Each question carries 2 *marks.*

13. Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of vitamin A per gram and 12 units of vitamin B and costs 20 paise per gram ; The daily minimum requirements of vitamin A and vitamin B are 100 units and 120 units respectively. Formulate this as a Linear programming problem to find the minimum cost of product mix.
14. Show that the set $S = \{(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \leq 4\} \subset R^3$ is a convex set.
15. Plot the feasible region in x_1, x_2 plane for the LPP constraints :

    $3x_1 - 2x_2 \leq 12 ; x_1 - 6x_2 \leq 1 ; - x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$.
16. State the characteristics of canonical form and write the canonical form of LPP in matrix form.
17. The column vector (1, 1, 1) is a feasible solution to the system of equations :

    $x_1 + x_2 + 2x_3 = 4$ and $2x_1 - x_2 + x_3 = 2$. Reduce the given feasible solution to a basic feasible solution.
18. Verify Minimax theorem for the function $f(x) = \{9, 7, 5, 3, 1\}$.
19. State the general rules for converting any primal LPP into its dual.
20. Explain the North- West corner rule for obtaining an initial basic feasible of a transportation problem.
21. Prove that every loop in a transportation table has an even number of cells.
22. What are the chief characteristics of a transshipment problem.
23. Write steps for solving assignment problem by Hungarian method.
24. In an assignment problem with cost (c_{ij}), if all $c_{ij} > 0$, then prove that feasible solution (x_{ij}) which satisfies $\sum\sum c_{ij} x_{ij} = 0$ is optimal.

(9 × 2 = 18 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 5 marks.

25. Show that set of all convex combinations of a finite number of vectors $x_1, x_2, \ldots x_k$ in R^n is a convex set.

26. Use graphical method to solve the LPP : Maximize $z = 5x_1 + 7x_2$ subject to the constraints,

$$x_1 + x_2 \leq 4$$
$$3x_1 + 8x_2 \leq 24$$
$$10x_1 + 7x_2 \leq 35$$
$$x_1, x_2 \geq 0.$$

27. Obtain all the basic feasible solution to the system of linear equations :

$$x_1 + 2x_2 + x_3 = 4 \text{ and } 2x_1 + x_2 + 4x_3 = 5.$$

28. Write the algorithm to solve LPP using Simplex method.

29. Use penalty method to :

Minimize $z = 2x_1 + x_2$

subject to the constraints, $3x_1 + x_2 = 3$

$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3 \text{ and } x_1, x_2, x_3 \geq 0.$$

30. Verify that dual of dual is primal for the following LPP :

Maximize $z = 2x_1 + 5x_2 + 6x_3$

subject to the constraints : $5x_1 + 6x_2 - x_3 \leq 3$

$$-2x_1 + x_2 + 4x_3 \leq 4$$
$$x_1 - 5x_2 + 3x_3 \leq 1$$
$$-3x_1 - 3x_2 + 7x_3 \leq 6$$
$$x_1, x_2, x_3 \geq 0.$$

31. Obtain an initial basic feasible solution to the following transportation problem using the matrix minima method.

	D_1	D_2	D_3	D_4	*Supply*
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
Required	4	6	8	6	24

Turn over

32. Prove that there always exist an optimal solution to a balanced transportation problem.

33. A company has 5 jobs to be done on five machines. Any job can be done on any machine. The cost of doing the jobs on different machines are given below. Assign the jobs for different machines so as to minimize the total cost.

	Machines				
Jobs	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

(6 × 5 = 30 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 10 *marks..*

34. Solve the following linear programming problem by simplex method :

Maximize $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

subject to the constraints :

$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$

$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$

$$7x_1 + x_4 \le 70 \text{ and } x_1, x_2, x_3, x_4 \ge 0.$$

35. Let X be set of column vectors of the co-efficient matrix of a transportation problem. Prove that a necessary and sufficient condition for vectors in X to be linearly dependent is that the set of their corresponding cells in the transportation contains a loop.

36. Determine the optimum basic feasible solution to the following transportation problem with the initial solution obtained by Vogel's approximation method :

	D_1	D_2	D_3	*Availability*
O_1	50	30	220	1
O_2	90	45	170	4
O_3	250	200	50	4
Demand	4	2	3	9

(2 × 10 = 20 marks)