D 32373 (Pages : 2) Name..................................

Reg. No..................................

FIRST SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2022

(CBCSS–UG)

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2020—2022 Admissions)

Time : Two Hours and a Half Maximum Marks : 80

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum marks that can be earned from this section is 25.

1. Draw the truth table of the disjunction of the proposition p and q.
2. Write down the negation of the statement with UD = the set of integers : $(\forall x)(\exists \mid x \mid = x)$.
3. State the prime number theorem.
4. Express (28, 12) as a linear combination of 28 and 12.
5. State the fundamental theorem of arithmetic.
6. Define Euler's phi function.
7. Show that $a \equiv b \pmod{m}$ if and only if $a = b + \text{km}$ for some integer k.
8. Briefly describe Euclidean algorithm.
9. Define the least common multiple of two positive integers.
10. What is a tautology ?
11. Prove that there is no positive integer between 0 and 1.
12. What do you mean by the well-ordering principle ?
13. Write down the recursive definition of the factorial function $f(n) = n!$
14. If $a \mid b$ and $b \rangle a$, show that $a = b$.
15. Show that $11 \times 14^n + 1$ is a composite number.

(Maximum ceiling 25 marks)

Turn over

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum marks that can be earned from this section is 35.

16. Distinguish between Converse, Inverse and Contrapositive of a proposition by giving example for each.

17. Prove that there is no polynomial $f(n)$ with integral co-efficients that will produce primes for all integers n.

18. Find the remainder when $1! + 2! + \ldots + 100!$ is divided by 15.

19. Show that $2^{2^5}+1$ is divisible by 641.

20. If p is a prime, then show that $(p-1)! \equiv -1 (\text{mod } p)$.

21. Solve the recurrence relation $h(n) = h(n-1) + (n-1)$ if $h(1) = 0$.

22. If a cock is worth five coins, a hen three coins, and three chicks together one coin, how many cocks, hens and chicks, totalling 100, can be bought for 100 coins ?

23. Show that the LDE $ax + by = c$ is solvable if and only if $d \mid c$, where $d = (a, b)$.

(Maximum ceiling 35 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 marks.
Maximum marks that can be earned from this section is 20.

24. (a) Conjecture a formula for the sum of the first n odd positive integers and then use induction to establish the conjecture.

    (b) Prove that no integer of the form $8n + 7$ can be expressed as a sum of three squares.

25. (a) State and prove Wilson's Theorem.

    (b) State and prove Euler's Theorem.

26. (a) Show that $a \equiv b (\text{mod } m)$ if and only if a and b leave the same remainder when divided by m.

    (b) Prove the divisibility criterion to test the divisibility by 11 and use it to check whether $n = 243, 506, 076$ is divisible by 11 or not.

27. (a) Define contrapositive of a statement and show that an implication is logically equivalent to its contrapositive by constructing the truth table.

    (b) Prove by contradiction that there is no largest prime.

(2 × 10 = 20 marks)