**D 140845** **(Pages : 4)** **Name..................................**

**Reg. No...............................**

**FOURTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION**
**APRIL 2026**

Mathematics

MTS4B04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half | Maximum : 80 Marks

**Section A**

*Short Answer Type Questions.*
*Each question carries 2 marks.*

1. Show that the system of linear equations $\begin{aligned} x + \ \ y &= 4 \\ 3x + 3y &= 6 \end{aligned}$ has no solution.

2. State free variable theorem for Homogeneous Systems. Which type of homogeneous linear system is having infinitely many solutions ?

3. Solve the matrix equation for $a, b, c$ and $d$, $\begin{bmatrix} a & 3 \\ -1 & a+b \end{bmatrix} = \begin{bmatrix} 4 & d-2c \\ d+2c & -2 \end{bmatrix}$.

4. Show that the unit vectors $i = (1, 0, 0)$, $j = (0, 1, 0)$ and $k = (0, 0, 1)$ span $R^3$.

5. Determine whether the vectors $(-3, 0, 0), (5, -1, 2), (1, 1, 3)$ in $R^3$ are linearly independent or not ?

6. Find the co-ordinate vector of $w = (1, 0)$ relative to the basis $s = [\bar{u}_1, \bar{u}_2]$ of $R^2$, where $\bar{u}_1 = (1, -1)$ and $\bar{u}_2 = (1, 1)$.

7. Give a solution of the change of Basis Problem.

8. Define the null space of A, where A is an $m \times n$ matrix. When you can say that the system of linear equations $Ax = b$ is consistent.

**Turn over**

9. Define rank of a matrix A and show that rank (A) = rank ($A^T$).

10. Find the reflection of the vector $x = (1, 5)$ about the line through the origin that makes an angle of $\frac{\pi}{6}$.

11. Find the image of the line $y = 4x$ under multiplication by the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

12. Confirm by multiplication that $x$ is an eigen vector of A and find the corresponding eigen value if $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

13. Let $R^2$ have the Weighted Euclidean inner product $\langle u, v \rangle = 2u_1 v_1 + 3u_2 v_2$.

    For $u = (1, 1)$ $v = (3, 2)$ $k = 3$ compute $\| u - kv \|$.

14. Let $P_2$ have the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x) q(x) \, dx$. For $p = x, q = x^2$, calculate $\| p \|$ and $\| q \|$.

15. If A is an $n \times n$ orthogonal matrix, then show that $\| Ax \| = \| x \|$ for all $x$ in $R^n$.

(Maximum 25 marks)

## Section B

*Paragraph | Problem Type Questions.*

*Each question carries 5 marks.*

16. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$ solve the system.

17. Define symmetric matrices. If A is an invertible symmetric matrix, then show that :

    (i) $AA^T$ and $A^TA$ symmetric.

    (ii) $A^{-1}$ is symmetric.

18. Define Wronskiann of the functions $f_1 = f_1(x), f_2 = f_2(x), \ldots f_n = f_n(x)$ which are $n-1$ times differentiable in $(-\infty, \infty)$. Use this to show that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $c^\infty(-\infty, \infty)$.

19. If W is a subspace of a finite dimensional vector space V, then show that :

    (i) dim W $\leq$ dim V.

    (ii) dim W = dim V implies V = W.

20. Consider the basis $B = [u_1, u_2]$ and $B^1 = [u_1^1, u_2^1]$ for $R^2$ where $u_1(2, 2)$ $u_2(4, -1)$ $u_1^1 = (1, 3)$ $u_2^1 = (-1, -1)$.

    (a) Find the transition matrix from $B^1$ to B.

    (b) Find the transition matrix from B to $B^1$.

21. If $T_1 : R^2 \to R^2$ is the reflection about $y$-axis and $T_2 : R^2 \to R^2$ is the reflection about $x$-axis, then show that both $T_1 \circ T_2$ and $T_2 \circ T_1$ are reflection about origin. Also explain this using standard matrices of $T_1$ and $T_2$.

22. Corresponding to the vectors in $R^3$ by $V_1 = (0, 1, 0), V_2 = (1, 0, 1)$ and $V_3 = (1, 0, -1)$, construct an orthonormal set.

23. Define eigen vector corresponding to an eigen value $\lambda$ of a square matrix A. if A is symmetric, show that eigen vectors corresponding to different eigen values are orthogonal.

(Maximum 35 marks)

**Turn over**

## Section C

*Essay Type Questions.*

*Each carry* 10 *marks : Answer any* **two** *questions.*

24\. (a) Show that every elementary matrix is invertible and the inverse is also an elementary matrix.

(b) What condition must $b_1, b_2$ and $b_3$ satisfy in order for the system of equations $x_1 + x_2 + 2x_3 = b_1$, $x_1 + x_3 = b_2$, $2x_1 + x_2 + 3x_3 = b_3$ to be consistent.

25\. (a) Let V be a vector space and $\bar{u}$ a vector in V and $k$ a scalar. Then show that :

(i) $o\bar{u} = 0$.

(ii) $(-1)\,\bar{u} = -\,\bar{u}$.

(b) Consider the basis $s = \{v_1, v_2, v_3\}$ where $v_1 = (1, 2, 1)$ $v_2 = (2, 9, 0)$ $v_3 = (3, 3, 4)$ of $R^3$. Find the co-ordinate vector of $v = (5, -1, 9)$ relative to the basis $s$. Also find the vector $v$ in $R^3$ whose co-ordinate vector relative to $s$ is $(-1, 3, 2)$.

26\. (a) If A is a matrix with $n$ columns, then establish a relationship between rank A and nullities of A.

(b) If A is the matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, then find a basis for the row space consisting of entirely row vectors from A.

27\. When you can say that a square matrix A is diagonalizable ? If A is an $n \times n$ matrix, show that the following statements are equivalent ?

(a) A is diagonalizable.

(b) A has $n$ linearly independent eigen vectors.

(20 marks)