D 70971 (Pages : 2) Name.....

Reg. No.....

THIRD SEMESTER M.Sc. DEGREE (REGULAR) EXAMINATION NOVEMBER 2019

(CUCSS)

Mathematics

MT 3C 13—COMPLEX ANALYSIS

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Show that any analytic function satisfies Cauchy-Riemann equations.
2. Show that Mobius transformations preserve cross ratios.
3. Find the ilnear transformations that maps 0, i, $-i$ into 1, – 1, 0, respectively.
4. Define $r:[0,2\pi]\to\mathbb{C}$ by $r(t)=e^{\text{int}}$, where n is some integer (positive, negative or zero). Show that $\int_r \frac{1}{z}dz = 2\pi in$.
5. If $f(z)$ and $g(z)$ have algebraic orders h and k at $z = a$, show that fg has the order $h + k$.
6. Determine the nature of the singularity of $e^{1/z}$ at $z = 0$.
7. State Rouche's theorem.
8. Find the residue of $\frac{e^z}{(z-a)^2}$ at $z = a$.
9. Prove that a bounded entire function is a constant.
10. If P(z) is a non-constant polynomial in $\mathbb{C}$, then prove that P(z) has a root in $\mathbb{C}$.
11. Develop tan(z) in powers of z upto the term containing z^4.
12. If $f(z)=\frac{1}{z(z-1)(z-2)}$, give Laurent series expansion of f in the annulus region $1 < |z| < 2$.
13. Show that if f is analytic is a region G and a is a point with $|f(a)| \geq |f(z)|$ for all z in G, then f is a constant function.
14. State Argument principle.

(14 × 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Define cross ratio and show that it is invariant under a linear transformation.
16. State and prove the symmetry principle.
17. Find an analytic function $f(z) = u + iv$ whose real part $u = e^x(x\cos y - y\sin y)$.
18. Suppose $f(z)$ is analytic in the region Ω obtained by omitting a point a from the region Ω. Prove that a is a removable singularity of $f(z)$ if $\lim_{z\to a}(z-a)f(z) = 0$.
19. State and prove Cauchy's integral theorem for an analytic function.
20. State and prove Morera's theorem.
21. Show that the sum of residues of an elliptic function is zero.
22. By the method of residues, evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta}\,d\theta$.
23. How many roots of the equation $z^4 + 9z^3 + 3z^2 + 8z + 3 = 0$ lie in the right half plane.
24. Let f be analytic in $B(a\,;R)$. Show that f has a power series expansion in $B(a;R)$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Prove that any two bases of the same module are connected by a unimodular transformation.
26. Show that if f is analytic on a region G and if $|f|$ attains a maximum on G, then f is a constant on G.
27. State and prove Laurent's series expansion for an analytic function $f(z)$ in an annulus $R_1 < |z-a| < R_2$.
28. Prove that the addition theorem for the Weierstrass $\mathbb{P}$-function :

$$\mathbb{P}(z+u) = -\mathbb{P}(z) - \mathbb{P}(u) + \frac{1}{4}\left(\frac{\mathbb{P}'(z) - \mathbb{P}'(u)}{\mathbb{P}(z) - \mathbb{P}(u)}\right)^2.$$

(2 × 4 = 8 weightage)