

D 132450

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Name.....

Reg. No.....

**FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION, NOVEMBER 2025**

Mathematics

MAT1CJ101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours

Maximum : 70 Marks

**Section A***All questions can be answered.**Each question carries 3 marks.**(Ceiling 24 marks).*

1. Draw the graph of the equation  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ .
2. Find the domain of the function  $f(x) = \frac{1}{x(x-1)(x-2)}$  in real numbers.
3. Find  $\lim_{x \rightarrow 0^+} \ln \frac{2}{x^2}$ .
4. Discuss the continuity of the function  $\sqrt{2x+3}$ .
5. Find  $\frac{dw}{dt}$  if  $w = \tan x$  and  $x = 4t^3 + t$ .
6. If  $y = \sin(3x)$ , then find  $\frac{d^2y}{dx^2}$ .
7. Find the absolute maximum and minimum values of  $f(x) = (x-2)^3$  on the closed interval  $[1, 4]$ .

**Turn over**

8. Verify that the hypotheses of the Mean-Value Theorem for the function  $f(x) = x^2 - x$  on the interval  $[-3, 5]$  and find all values of  $c$  in that interval that satisfy the conclusion of the theorem.
9. Find the relative extrema of the function  $f(x) = \frac{x+1}{x-1}$ .
10. Give a graph of the function  $f(x) = x + \sin x$  and identify the locations of all relative extrema and inflection points.

### Section B

*All questions can be answered.*

*Each question carries 6 marks.*

*(Ceiling 36 marks)*

11. If  $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 3$ , then find  $\lim_{x \rightarrow 2} f(x)$ .
12. Find  $\lim_{x \rightarrow 4} \frac{3 - x}{x^2 - 2x - 8}$ .
13. If functions  $f(x)$  and  $g(x)$  are continuous for  $0 \leq x \leq 1$ , could  $f(x)/g(x)$  possibly be discontinuous at a point of  $[0, 1]$ ? Give reasons for your answer.
14. Use implicit differentiation to find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ , if  $\sqrt{y} = 3x^2 + \tan x$ .
15. Find the absolute extrema values of  $F(t) = 8t - t^4$  on  $[-2, 1]$ .
16. Find the critical points of  $f(x) = x^{1/3}(x - 4)$ . Identify the intervals on which  $f$  is increasing and decreasing. Find the function's local and absolute extreme values.

17. Find the asymptotes of the curve  $f(x) = 2 + \frac{\sin x}{x}$ .

18. Solve  $\lim_{x \rightarrow -\infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$ .

### Section C

Answer any **one** questions.

Each question carries 10 marks.

19. At time  $t \geq 0$ , the velocity of a body moving along the  $s$ -axis is  $v = t^2 - 4t + 3$ .

(a) Find the body's acceleration each time the velocity is zero.

(b) When is the body moving forward? moving backward?

(c) When is the body's velocity increasing? decreasing?

20. (a) Show that  $f(x) = x^3 + \frac{4}{x^2} + 7$  has exactly one zero in the interval  $(-\infty, 0)$ .

(b) Suppose the derivative of the function  $y = f(x)$  is  $y' = (x - 1)^2(x - 2)(x - 4)$ . At what points, if any, does the graph of  $f$  have a local minimum, local maximum, or point of inflection?

(1 × 10 = 10 marks)