

**FIRST SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION  
NOVEMBER 2020**

Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

**Part A (Objective Type Questions)**

*Answer all questions (1 - 12).*

*Each question carries 1 mark.*

1.  $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = \dots$
2.  $\lim_{x \rightarrow 0} \frac{\sin(2+x) - \sin 2}{x} = \dots$
3. Define removable discontinuity.
4. State the condition(s) for local maximum of the function  $y = f(x)$ .
5. What is (are) the vertical asymptote(s) of the curve  $xy^3 - 2xy^2 - 2y^3 - 4 = 0$ .
6. State Rolle's theorem.
7. Find  $\frac{d}{dx}(\cosh(3x-2))$ .
8. State the second derivative test for concavity of a function  $y = f(x)$ .
9. State the mean value theorem for definite integral.
10.  $\sum_{k=1}^4 (k^2 - 3k) = \dots$
11. Let  $f$  be a continuous function on  $[a, b]$ . Then what is the average value of  $f$  on  $[a, b]$ .
12. Area bounded by the curves  $y = f(x), y = g(x)$  and the ordinates  $x = a$  and  $x = b$  is given by \_\_\_\_\_.

(12 × 1 = 12 marks)

**Turn over**

**Part B (Short Answer Type)**

Answer any **nine** questions (13 - 24).

Each question carries 2 marks.

13. If  $\sqrt{3-2x} \leq f(x) \leq \sqrt{3-x}$ , find  $\lim_{x \rightarrow 0} f(x)$ .
14. Find  $\lim_{x \rightarrow 2} \frac{x-3}{x^2-4}$ .
15. Find the equation of the tangent line to the curve  $y = \sqrt{x}$  at  $x = 4$ .
16. Find the absolute extrema of  $f(x) = x^{2/3}$  on  $[-2, 3]$ .
17. Find the points of inflection of the function  $y = 2 + \cos x, x \geq 0$ .
18. Find  $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$ .
19. Find the horizontal asymptotes of the graph of the function  $f(x) = \frac{-8}{x^2 - 4}$ .
20. Find the linearization of  $f(x) = x^3 - 2x + 3$  at  $x = 2$ .
21. Find  $\frac{dy}{dx}$  if  $y = \int_1^{x^2} \cos t \, dt$ .
22. Find  $\lim_{x \rightarrow 1} \frac{1-x}{\log x}$ .
23. Find  $\lim_{x \rightarrow \infty} x^{1/x}$ .
24. Verify Rolle's theorem for the function  $f(x) = \tan x$  in  $[0, \pi]$ .

(9 × 2 = 18 marks)

**Part C (Short Essay Type)**

Answer any **six** questions (25 - 33).

Each question carries 5 marks.

25. State and prove the rule for the limit of a sum.
26. Show that if a function  $f$  has a derivative at  $x = c$ , then show that  $f$  is continuous at  $x = c$ .
27. State and prove Rolle's theorem.

28. Verify mean value theorem for the function  $f(x) = x^3 - 3x^2 + 2x$  in  $\left[0, \frac{1}{2}\right]$ .
29. Find the intervals on which  $f(x) = -x^3 + 12x + 5, x \in [-3, 3]$  is increasing and decreasing.
30. Find all the asymptotes of  $f(x) = \frac{x^2 - 3}{2x - 4}$ .
31. Give an example of a function which is not Riemann integrable. Prove your claim.
32. Find the area between  $y = \sec^2 x$  and  $y = \sin x$  from 0 to  $\pi/4$ .
33. Verify the mean value theorem for integrals for the function  $f(x) = \frac{x}{\sqrt{x^2 + 16}}$  in  $[0, 3]$ .

(6 × 5 = 30 marks)

**Part D (Essay Questions)**Answer any **two** questions (34 - 36).

Each question carries 10 marks.

34. A dynamite blast blows a heavy rock straight up with a velocity of 160 ft/sec. It reaches a height of  $s = 160t - 16t^2$  ft after  $t$  seconds.
- How high does the rock go ?
  - What is the velocity and speed of the rock when it is at 256 ft above the ground on the way up? on the way down ?
  - What is the acceleration of the rock at any time  $t$  during its flight ?
35. Sketch the graph of the function  $y = x^4 - 4x^3 + 10$ , by inspecting increasing, decreasing, concavity, points of inflection, local extrema etc.
36. a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at  $45^\circ$  angle at the center of the cylinder. Find the volume of the wedge by slicing method.
- b) Find the area of the region bounded by the curves  $y = x^2$  and  $y = x^4 - 4x^2 + 4$ .

(2 × 10 = 20 marks)