

C 82882

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Name.....

Reg. No.....

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION  
JUNE 2020

(CUCSS)

Mathematics

MT 2C 09—ODE AND CALCULUS OF VARIATIONS

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

**Part A**

*Answer all questions.*

*Each question carries 1 weightage.*

1. Write  $\sin x$  in power series form.
2. Determine the nature of the point  $x = 0$  for the equation  $y'' + (\sin x)y = 0$ .
3. Find the Legendre polynomial  $P_2(x)$  using Rodrigue's formula.
4. Write geometric series in the hypergeometric series form.
5. If  $y_{\frac{1}{2}}(x)$  be a non-trivial solution of Bessel's equation on the positive  $x$  axis, then find the distance between successive zeros of  $y_{\frac{1}{2}}(x)$ .
6. True or False :  $P_n(x) \leq P_{n+1}(x)$ ,  $x > 1$ , where  $P_n(x)$  is the Legendre polynomial of degree  $n$ .
7. If  $1 + x^{10} = \sum_{n=0}^{10} C_n P_n(x)$ , then find  $C_5$ , where  $P_n(x)$  be the Legendre polynomial of degree  $n \geq 0$ .

Turn over

8. Find the critical point of the system  $\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$ .
9. What are the eigenvalues of the problem  $y'' + \lambda y = 0$ ,  $y(0) = y(1) = 0$ .
10. What will be nature of critical point, if the roots  $m_1$  and  $m_2$  of the auxiliary equation are conjugate complex but not purely imaginary?
11. Sketch the phase portrait of the system  $\begin{cases} \frac{dx}{dt} = -y \\ \frac{dy}{dt} = x \end{cases}$ .
12. For what conditions on  $a, b, c$ , the Liapunov function  $E(x, y) = ax^2 + bxy + cy^2$  is positive definite?
13. Write Euler-Lagrange equation.
14. Write the normal form of Bessel's equation.

(14 × 1 = 14 weightage)

**Part B**

*Answer any seven questions.  
Each question carries 2 weightage.*

15. Find the radius of convergence of the series  $\sum_{n=0}^{\infty} \frac{x^n}{n^n}$  using ratio test.
16. Determine the nature of points  $x = 0, 1$  for the differential equation  $x^2(1-x)y'' + xy' + y = 0$ .
17. Verify  $\frac{d}{dx} [x^p J_p(x)] = x^p J_{p-1}(x)$ .
18. Find the first approximation of the initial value problem  $y' = y$ ,  $y(0) = 1$ .

19. Write the indicial equation for  $x(1+x^2)y'' + (\cos x)y' + (1-3x+x^2)y = 0$ .
20. Show that  $\Gamma(p+1) = p\Gamma(p)$ .
21. State Sturm separation theorem.
22. Show that the solution  $u(x)$  of the equation  $u''(x) + xu(x) = 0$ ,  $x > 0$  has infinitely many zeros on the positive  $x$ -axis.
23. Define geodesic problem.
24. Find the extremal for the functional  $\int_{x_1}^{x_2} (y^2 - y'^2) dx$ .

(7 × 2 = 14 weightage)

**Part C***Answer any two questions.**Each question carries 4 weightage.*

25. Express  $f(x) = x^3 + 2x^2 - x - 3$  in terms of Legendre polynomials.
26. If  $q(x) < 0$  and  $u(x)$  is a non-trivial solution of  $u'' + q(x)u = 0$  then prove that  $u(x)$  has at most one zero.
27. Find the general solution of  $\begin{cases} \frac{dx}{dt} = 4x - y \\ \frac{dy}{dt} = 2x + y \end{cases}$ .
28. Find the curve joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  that yields a surface of revolution of minimum area when revolved about the  $x$ -axis.

(2 × 4 = 8 weightage)