

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2018

(CUCSS-PG)

Mathematics

MT 2C 11—OPERATIONS RESEARCH

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A*Answer all questions.**Each question carries 1 weightage.*

1. Prove that the linear function $f(X) = CX$, $X \in E_n$ is both concave and convex.
2. Prove that the set S_F of feasible solutions, if not empty, is a closed convex set bounded from below and so has at least one vertex.
3. Why do we introduce new variables in Linear Programming Problems ?
4. What is canonical form of equations ?
5. Prove that the dual of the dual is the primal in linear programming problems.
6. Describe the various applications of linear programming.
7. What are simplex multipliers ?
8. Describe the fixed charge problem in integer programming.
9. Describe unbalanced transportation problem.
10. What is Caterer problem in operations research ?
11. Illustrate branch and bound method through an example.
12. Describe the terms : chain, path, cycle, circuit and component with reference to graphs.
13. Define Mathematical expectation in game theory. Illustrate by an example.
14. Describe the notion of dominance in game theory.

(14 × 1 = 14 weightage)

Part B*Answer any seven questions.**Each question carries 2 weightage.*

15. Let $K \subset E_n$ be a convex set, $X \in K$, and $f(X)$ a convex function. If $f(X)$ has a relative minimum, then prove that it is also a global minimum.

Turn over

16. Solve graphically the linear programming problem :

Maximize $4x_1 + 5x_2$ subject to $x_1 - 2x_2 \leq 2, 2x_1 + x_2 \leq 6, x_1 + 2x_2 \leq 5, -x_1 + x_2 \leq 2,$
 $x_1 + x_2 \geq 1, x_1 \geq 0, x_2 \geq 0.$

17. Prove that if the primal problem is feasible, then it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa.
18. Prove that the transportation problem has a triangular basis.
19. Describe the concept of cutting planes in integer programming.
20. Describe the minimum path problem in network analysis.
21. Prove that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.
22. Describe the two-person, zero-sum game.
23. Explain the terms mixed strategy, pure strategy and optimal strategies with reference to any matrix game.
24. State and prove the mini max theorem in theory of games.

(7 × 2 = 14 weightage)

Part C

Answer any two questions.

Each question carries 4 weightage.

25. Let $f(X)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Prove that $f(X)$ is a convex function if and only if $f(X_1) - f(X_2) \geq (X_2 - X_1)' \nabla f(X_1)$ for all $X_1, X_2 \in K$.
26. By Simplex method, minimize $f(X) = 4x_1 + 5x_2$; subject to $2x_1 + x_2 \leq 6, x_1 + 2x_2 \leq 5, x_1 + x_2 \geq 1,$
 $x_1 + 4x_2 \geq 2, x_1, x_2 \geq 0.$
27. Solve that the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the following table. Obtain three optimal solutions :

	D ₁	D ₂	D ₃	D ₄	
O ₁	1	2	-2	3	70
O ₂	2	4	0	1	38
O ₃	1	2	-2	5	32
	40	28	30	42	

28. Solve the game where the payoff matrix is :

$$\begin{pmatrix} 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

(2 × 4 = 8 weightage)