

C 21545

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Name.....

Reg. No.....

FOURTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer Type Questions)*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

- Show that the linear system of equations $4x - 2y = 1$ has infinitely many solutions.
 $16x - 8y = 4$
- Write any two facts about row echelon forms and reduced row echelon forms.
- Express the linear system

$$\begin{aligned} 4x_1 - 3x_3 + x_4 &= 1 \\ 5x_1 + x_2 - 8x_4 &= 3 \\ 2x_1 - 5x_2 + 9x_3 - x_4 &= 0 \\ 3x_2 - x_3 + 7x_4 &= 2 \end{aligned}$$
 in the form $AX = B$.
- Let $V = \mathbb{R}^2$ and define addition and scalar multiplication as follows. For $\bar{u} = (u_1, u_2), \bar{v} = (v_1, v_2)$,
 $\bar{u} + \bar{v} = (u_1 + v_1, u_2 + v_2)$ and for a real number $k, k\bar{u} = (ku_1, 0)$. For $\bar{u} = (1, 1)$ and $\bar{v} = (-3, 5)$ find
 $\bar{u} + \bar{v}$ and for $k = 5$, find $k\bar{u}$. Also show that one axiom for vector space is not satisfied.
- Define basis for a vector space.
- How will you relate the dimension of a finite dimensional vector space to the dimension of its subspace. Give two facts.
- Give a solution to the change of basis problem.
- When you can say that a system of linear equation $Ax = b$ is consistent. What is meant by a particular solution of the consistent system $Ax = b$.
- Find the rank of a 5×7 matrix A for which $Ax = 0$ has a two-dimensional solution space.

Turn over

10. If $T_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is a matrix transformation. Then define its kernel $\ker(T_A)$ and Range of (T_A) . What is $\ker(T_A)$ in terms of null-space of A .
11. Discuss the geometric effect on the unit square of multiplication by a diagonal matrix $A = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$.
12. Confirm by multiplication that x is an eigen vector of A and find the corresponding eigen value, if $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
13. Let \mathbb{R}^2 have the weighted Euclidean inner product $\langle u, v \rangle = 2u_1v_1 + 3u_2v_2$. For $u = (1, 1), v = (3, 2)$, compute $d(u, v)$.
14. If u and v are orthogonal vectors in a real inner product space, then show that $\|u+v\|^2 = \|u\|^2 + \|v\|^2$.
15. State four properties of orthogonal matrices.

(10 × 3 = 30 marks)

Section B (Paragraph/ Problem Type Questions)*Answer at least five questions.**Each question carries 6 marks.**All questions can be attended.**Overall Ceiling 30.*

16. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form

$$\text{as } \begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix} \text{ solve the system.}$$

17. If A is an invertible matrix, then show that A^T is also invertible and $(A^T)^{-1} = (A^{-1})^T$.
18. Let V be a vector space and \bar{u} , a vector in V and k a scalar. Then show that (i) $0\bar{u} = 0$;
(ii) $(-1)\bar{u} = -\bar{u}$.

19. If $S = [v_1, v_2, \dots, v_n]$ is a basis for a vector space V , then show that every vector v in V can be expressed in form $v = c_1v_1 + c_2v_2 + \dots + c_nv_n$ in exactly one way. What are the co-ordinates of V relative to the basis S ?
20. Consider the basis $B = [u_1, u_2]$ and $B' = [u'_1, u'_2]$ for \mathbb{R}^2 , where $u_1 = (2, 2)$ $u_2 = (4, -1)$
 $u'_1 = (1, 3)$ $u'_2 = (-1, -1)$.
- (a) Find the transition matrix from B' to B .
- (b) Find the transition matrix from B to B' .
21. If A is a matrix with n columns, then define rank A , nullity of A and establish a relationship between them.
22. Define eigen space corresponding to an eigen value λ of a square matrix A . Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.
23. Use the Gram-Schmidt process for an orthonormal basis corresponding to the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (0, 1, 1)$ and $u_3 = (0, 0, 1)$.

(5 × 6 = 30 marks)

Section C (Essay Type Questions)

*Answer any two questions.
Each question carries 10 marks.*

24. Show that the following statements are equivalent for an $n \times n$ matrix A :
- (a) A is invertible.
- (b) $Ax = 0$ has only the trivial solution.
- (c) The reduced row echelon form of A is I_n .
- (d) A is expressible as a product of elementary matrices.
25. (a) Define Wronskian of the functions $f_1 = f_1(x), f_2 = f_2(x) \dots f_n = f_n(x)$ which are $n - 1$ times differentiable in $(-\infty, \infty)$. Use this to show that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $C^\infty(-\infty, \infty)$.
- (b) Show that the vectors $v_1 = (1, 2, 1), v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$ form a basis for \mathbb{R}^3 .

Turn over

26. (a) If A is the matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, then find a basis for the row space consisting entirely

row vectors from A .

- (b) Find the standard matrix for the operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ that first rotates a vector counter clockwise about z -axis through an angle θ , reflects the resulting vector about yz plane and then projects that vector orthogonally onto the xy plane.

27. (a) On P_2 , polynomial in $[-1,1]$, define innerproduct as $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$. Find $\|p\|, \|q\|$

and $\langle p, q \rangle$ for $p = x$ and $q = x^2$.

- (b) If A is an $n \times n$ matrix with real entries, show that A is orthogonally diagonalizable if and only if A has an orthonormal set of n eigenvectors.

(2 × 10 = 20 marks)