

D 12034

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Name.....

Reg. No.....

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019–2020 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A

*Answer at least eight questions.
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.*

1. Evaluate $\int_0^1 (t\hat{i} + 3t^2\hat{j} + 4t^3\hat{k}) dt$.
2. The position of a moving particle is $\vec{r}(t) = t^2\hat{i} + t\hat{j} + t^3\hat{k}$. Find velocity and acceleration of the particle at $t = 2$.
3. If $z = e^{-y} \cos x$ find $\frac{\partial^2 z}{\partial x \partial y}$.
4. Find the level surface of $F(x, y, z) = x^2 + y^2 + z^2$ passing through $(1, 1, 1)$.
5. Evaluate $\oint_C x dx$, where C is the circle $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$.
6. Show that $\text{curl } \vec{r} = \vec{0}$.
7. State Green's theorem in the plane.
8. Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz \, dx \, dy \, dz$.
9. Write the equation of the circle with centre $(1, 2)$ and radius 4 in the complex plane.

Turn over

10. Find the value of i^{2i} .

11. Evaluate $\oint_C \frac{ze^z}{(z-3)^2} dz$, where C is $|z|=2$.

12. Evaluate $\oint_C \frac{dz}{z}$, where C is $|z|=1$.

(8 × 3 = 24 marks)

Section B

*Answer at least five questions.
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.*

13. Use chain rule to find $\frac{dw}{dx}$ at (0, 1, 2) for $w = xy + yz$; $x = \cos x$, $y = \sin x$, $z = e^x$.

14. Find the directional derivative of $f(x, y) = \sqrt{x^2y + 2y^2z}$ at (-2, 2, 1) in the direction of the negative z-axis.

15. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$ using double integrals.

16. Use polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5+x^2+y^2} dy dx$.

17. Show that $f(z) = (2x^2 + y) + i(y^2 - x)$ is not analytic at any point.

18. Evaluate $\oint_C \frac{5z+7}{z^2+2z-3} dz$, where C is the circle $|z-2|=2$.

19. Evaluate $\int \operatorname{Re} z dz$ along a line segment from $z=0$ to $z=1+2i$.

(5 × 5 = 25 marks)

Section C

Answer any **one** question.
The question carries 11 marks.

20. Let $\vec{F}(x, y, z) = z\hat{j} + z\hat{k}$ represents the flow of a liquid. Find the flux of \vec{F} through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward.
21. Use triple integrals to find the volume of the solid with in the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

(1 × 11 = 11 marks)