

D 30572

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Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION  
NOVEMBER 2022**

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

**Section A**

*Answer any number of questions.*

*Each question carries 2 marks. Ceiling is 20.*

1. Does the equation  $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$  represents a hyperbola? Justify your answer.
2. Find the vertex and directrix of the parabola  $y^2 = 4x$ .
3. Show that the parametric equation  $x = 3 \cos t, y = 2 \sin t; -\pi < t \leq \pi$  represents the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ .
4. State the reflection property of the parabola.
5. Find the remainder when  $f(x) = x^7 - 7x^3 + 1$  is divisible by  $x + 1$ .
6. Find the sum of the squares of the roots of the equation  $x^4 - 2x + 1 = 0$ .
7. State the Fundamental Theorem of Algebra.
8. Solve the equation  $(a - b)x^2 - (b - c)x + (c - a) = 0$ .
9. Find  $\Delta$  of the equation  $x^3 + 10x - 7 = 0$ .
10. Show that  $\sqrt[3]{\sqrt{5} + 2} - \sqrt[3]{\sqrt{5} - 2} = 1$ .

**Turn over**

11. Show that the equation  $x^6 - x^4 + 4x - 11 = 0$  has a root lies between 1 and 2.
12. Find the real root of the cubic equation  $x^3 + 9x - 2 = 0$ .

### Section B

*Answer any number of questions.*

*Each question carries 5 marks. Ceiling is 30.*

13. (i) State the Fundamental theorem of Affine Geometry.
- (ii) Determine the affine transformation which maps the points  $(0, 0)$ ,  $(1, 0)$  and  $(0, 1)$  to the points  $(3, 2)$ ,  $(5, 8)$  and  $(7, 3)$  respectively.
14. Determine the image of the line  $y = -x$  under the affine transformation  $t(x) = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} x + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ .
15. Solve  $x^3 - 9x^2 + 26x - 24 = 0$  if the roots form an arithmetic progression  $\alpha - \beta, \alpha, \alpha + \beta$ .
16. Factorize into real linear and quadratic factors of the polynomial  $f(x) = x^4 + 9$ .
17. Find the rational roots of the equation  $4x^3 - 4x^2 - x + 1 = 0$ .
18. Separate the roots of the equation  $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$ .
19. Show that the necessary and sufficient condition for an equation  $x^3 + px + q = 0$  to have three real and distinct roots is  $4p^3 + 27q^2 < 0$ .

### Section C

*Answer any one question.*

*The question carries 10 marks.*

20. (i) Prove that a perpendicular from a focus of a parabola to a tangent meets the tangent on the directrix of the parabola.
- (ii) Determine the equation of the tangent to the ellipse with parametric equations  $x = 3 \cos t$ ,  $y = \sin t$  at the point with parameter  $t = \pi/4$ . Deduce the co-ordinates of the point of intersection of this tangent with the  $x$ -axis.
21. Solve the cubic equation  $x^3 - 3x^2 + 12x + 16 = 0$  by using Carden's formula.

(1 × 10 = 10 marks)