D 12033 **(Pages : 3)** **Name..............................**

Reg. No..............................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE – 2

(2019–2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. Determine whether the function $f(x) = x^3 - 3x + 1$ has an inverse.

2. Find the derivatives of (a) $3^{\sqrt{x}}$; (b) $\cos^{-1}(3x)$.

3. Find the derivative of $\log\left[\dfrac{x^2(2x^2+1)^3}{\sqrt{5-x^2}}\right]$ when $x = 1$.

4. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$.

5. Find $\lim\limits_{x\to\infty}\dfrac{\log n}{n}$.

6. Determine whether the series converges. If it converges find the sum $\sum\limits_{n=1}^{\infty}\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$.

7. Use integral test to determine whether $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2+1}$ converges or diverges.

8. Show that the alternating series $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n}$ converges.

Turn over

9. Find the Maclaurin's series of $f(x) = \cos x$.

10. Find the radius of convergence and interval of convergence of the power series $\sum_{n=0}^{\infty} n!x^n$.

11. Describe the curve represented by $x = 4\cos\theta$ and $y = 3\sin\theta, 0 \le \theta \le 2\pi$.

12. Find the angle between the two planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

13. Find an equation in rectangular co-ordinates for the surface with the cylindrical co-ordinates $r^3 \cos 2\theta - z^2 = 4$.

14. Find a vector function that describes the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the plane $x + y + 2z = 4$.

15. Evaluate $\int_0^1 r(t)\, dt$ if $r(t) = t^2 i + \frac{1}{t+1} j + e^{-t} k$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 marks.
All questions can be attended.
Overall Ceiling 30.

16. Use logarithmic differentiation to find the derivative of $y = \sqrt[3]{\frac{x-1}{x^2+1}}$.

17. Find the derivative of $y = x^2 \operatorname{sech}^{-1}(3x)$.

18. Evaluate $\int_0^1 \log x\, dx$.

19. Show that the series $\sum_{n=1}^{\infty}\left[\frac{2}{n(n+1)} - \frac{4}{3^n}\right]$ is convergent and find its sum.

20. Find the tangent lines of $r = \cos 2\theta$ at the origin.

21. Find the length of the Cardioid $r = 1 + \cos\theta$.

22. Find the parametric equations for the line of intersection of the planes defined by $3x - y + 2z = 1$ and $2x + 3y - z = 4$.

23. Find the velocity vector, acceleration vector and speed of a particle with position vector :

$r(t) = \sqrt{t}\, i + tt^2 j + e^{2t} k, t \geq 0.$

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find the derivative of $\sec^{-1}\left(e^{-2x}\right)$.

(b) Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x}\right)^{\sin x}$.

25. (a) Find the area S of the surface obtained by revolving the circle $r = \cos\theta$ about the line $\theta = \pi/2$.

(b) Show that the surface area of a sphere of radius r is $4\pi r^2$.

26. (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ is convergent or divergent.

(b) Show that sequence $\left\{\frac{2^n}{n!}\right\}$ is convergent and find its limit.

27. (a) Find an equation in rectangular co-ordinates for the surface with spherical equation $\rho = 4\cos\phi$.

(b) A moving object has an initial position and an initial velocity given by the vectors $r(0) = i + 2j + k$ and $v(0) = i + 2k$. Its acceleration at time t is $a(t) = 6t\, i + j + 2k$. Find its velocity and position at time t.

(2 × 10 = 20 marks)