

D 130230

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Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
NOVEMBER 2025**

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Syllabus)

Time : Two Hours

Maximum : 60 Marks

Part A (Short Answer type)

All questions can be answered.

Each question carries 2 marks.

(Ceiling 20)

- Find the equation of the tangent at the point (2, 4) to the parabola $y^2 = 8x$.
- Write different steps in the classification of a conic E with equation
 $Ax^2 + Bxy + Cy^2 + Fx + Gy + H = 0$.
- Define Euclidean-congruent. True or False “Euclidean-congruence is an equivalence relation.”
- Determine whether the transformations of \mathbb{R}^2 , given by

$$t_1(x) = \begin{pmatrix} -6 & 5 \\ 3 & 2 \end{pmatrix} x + \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

is an affine transformation. Why ?

- By the method of detached co-efficients divide $x^6 - x^3 + 8x^3 + 25$ by $x^4 - 34x^2 + 2$.
- Without actual division show that $2x^7 + 2x^6 + x^4 + 2x^3 + x^2 + 2$ is divisible by $x^3 + 1$.
- State Identity Theorem.
- Find a lower limit of the negative roots of the equation

$$2x^6 + 20x^5 + 30x^3 + 50x + 1 = 0.$$

Turn over

9. Solve : $x^4 + 4x - 1 = 0$.
10. Verify that the following equation have roots in the intervals indicated :

$$x^3 - 3x^2 - 4x + 13 = 0. \text{ Roots in } (1, 8/3), (8/3, 3), (-3, -2)$$

11. State Rolles Theorem.
12. Separate the roots of the following equation:

$$x^4 - 4x^3 + 4x^2 - 24x - 1 = 0.$$

Part B (Paragraph/Problem Type)

All questions can be answered.

Each question carries 5 marks.

(Ceiling 30 marks)

13. Prove that the equation of the tangent at the point (x_1, y_1) to the rectangular hyperbola $xy = 1$ is

$$\frac{1}{2}(xy_1 + x_1y) = 1.$$

14. Determine the equation of the tangent at the point with parameter t to the ellipse with parametric equations :

$$x = a \cos t,$$

$$y = b \sin t,$$

where $t \in (-\pi, \pi], t \neq 0, \pi$.

15. Determine the image of the line $3x - y + 1 = 0$ under the affine transformation

$$t(x) = \begin{pmatrix} 1/2 & -1/2 \\ -1 & 2 \end{pmatrix} x + \begin{pmatrix} -1/2 \\ 4 \end{pmatrix}; x \in \mathbb{R}^2.$$

16. By Taylor's formula calculate the values of the following polynomial and their derivatives for the value of x indicated :

$$x^5 - 3x^2 + 4x_7 \text{ for } x = 2.$$

17. Examine for integral roots :

$$x^3 - 9x^2 + 22x - 24 = 0.$$

18. Solve the cubic equation :

$$2x^3 - 3x + 5 = 0.$$

19. How many real roots do the following equations have ?

$$x^6 + x^4 - x^3 - 2x - 1 = 0.$$

Part C (Essay Type)

*Answer any **one** of the following questions.*

Each question carries 10 marks.

20. Consider parabola with parametric equations $x = at^2, y = 2at$ ($t \in \mathbb{R}$).

- (a) Determine the equation of the chord joining the points P_1 and P_2 on the parabola with parameters t_1 and t_2 , respectively, where t_1 and t_2 are unequal and non-zero
- (b) Now assume that the chord P_1P_2 passes through the focus $(a, 0)$ of the parabola. Then prove that $t_1t_2 = -1$.

21. (a) Solve $x^3 - 2(1+i)x^2 - (1-2i)x + 2(1+2i) = 0$ given one root $1 + 2i$.

- (b) Find limits of the moduli of roots for the equations

$$6x^5 - 10x^4 + 7x^3 + 8x - 10 = 0.$$

(1 × 10 = 10 marks)