C 80708 (Pages : 3) Name..................................

Reg. No..................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCSS)

Mathematics

MT4 E07—ADVANCED FUNCTIONAL ANALYSIS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. Define dual basis with reference to a basis of a finite dimensional normed space.

2. What is the dual of $\mathbb{K}^n$ with norm $\| \|_p$.

3. Let X and Y be Banach spaces and $F \in BL(X, Y)$. If $R(F) = Y$ is bounded, then prove that F′ is bounded below.

4. Show that dual of a separable reflexive normed space is separable.

5. Define adjoint of a bounded operator on a Hilbert space.

6. What is meant by complemented subspace property.

7. Define uniformly continuous linear operator on an innerproduct space. Show that a bounded linear operator is uniformly continuous.

8. Let H be a Hilbert space and $A \in BL(H)$. Then show that $\|A\| = \|A^*\|$.

9. Define normal operator and unitary operator.

10. Define Fredholm integral operator on a Hilbert space.

11. If A and B are positive operators, prove that A + B is also a positive operator.

12. Let H be a Hilbert space and $A \in BL(X)$. If A is invertible, then prove that the adjoint A^* is invertible.

Turn over

13. Prove that the numerical range of an operator on a Hilbert space is a bounded subset of the field of scalars.

14. Define approximate eigenvalue of an operator.

(14 × 1 = 14 weightage)

Part B

Answer any **seven questions.**
Each question carries weightage 2.

15. Let X be a normed space. If X' is separable, then prove that X is separable.

16. Let X and Y be normed spaces. If $F_1, F_2 \in BL(X, Y)$, prove that $(F1 + F2)' = F_1' + F_2'$.

17. Let X be a Banach space and $A \in BL(X)$. Then with usual notations prove that $\sigma(A) = \sigma_a(A) \cup \sigma_e(A') = \sigma(A')$.

18. Let X be a reflexive normed space. Prove that every bounded sequence in X has a weak convergent subsequence.

19. Prove that a subset of a Hilbert space is weak bounded if and only if it is bounded.

20. Let H be a Hilbert space and $A \in BL(H)$. Prove that $\|A^* A\| = \|A\|^2 = \|AA^*\|$.

21. Let H be a Hilbert space and $A \in BL(H)$. Prove that A^* is injective if and only if the range space R (A) is dense in H.

22. Let H be a Hilbert space and $A, B \in BL(H)$ be normal operators. Then if A commutes with B^* and B commutes with A^*, prove that A + B and AB are normal.

23. Let H be a Hilbert space and $A \in BL(H)$. Prove that $k \in \sigma_a(A)$ if and only if $\bar{k} \in \sigma_a(A^*)$.

24. Let H be a Hilbert space and $A \in BL(H)$. Show that if A is compact, then A^* is also compact.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. Prove that the dual of c_{00} with the norm $\| \|_p$ is linearly isometric to l^q where $1/p + 1/q = 1$ and $1 \leq p \leq \infty$.

26. State and prove Riesz representation theorem for $C([a, b])$.

27. Let H be a non-zero Hilbert space and $A \in BL(H)$ be self adjoint. With usual notations, show that $\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subseteq [m_A, M_A]$.

28. Let H be a Hilbert space and $A \in BL(H)$ be Hilbert-Schmidt operator. Then prove that A is compact.

(2 × 4 = 8 weightage)