

D 112341

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Name.....

Reg. No.....

**FIRST SEMESTER (CUFYUGP) DEGREE EXAMINATION  
NOVEMBER 2024**

Mathematics

MAT 1CJ 101—DIFFERENTIAL CALCULUS

(2024 Admission onwards)

Time : Two Hours

Maximum Marks : 70

**Section A**

*All questions can be answered.  
Each question carries 3 marks.  
(Ceiling 24 marks)*

1. Draw the graph of the equation  $y^2 = 4x$ .
2. Write any three properties of absolute value function  $f(x) = |x|$ .
3. Find  $\lim_{x \rightarrow +\infty} \left( \frac{1 - e^x}{1 + e^x} \right)$ .
4. Does  $\lim_{x \rightarrow 1} \frac{1}{1-x} = \lim_{x \rightarrow 0^-} \frac{1}{x}$  exist ? Explain.
5. At what points are the function  $\frac{x}{\sin x}$  continuous ?
6. Let  $f(x) = x^2$  and  $g(x) = x + 6$ . Find  $(g \circ f)(x)$  and  $(g \circ f)'(x)$ .
7. Find the absolute maximum and minimum values of  $f(x) = 4x^2 - 12x + 10$  on the closed interval  $[1, 2]$ .
8. Find the two  $x$ -intercepts of the function  $f(x) = x^2 - 5x + 4$  and confirm that  $f'(c) = 0$  at some point  $c$  between those intercepts.
9. Locate the critical points of the function  $f(x) = x^3 - 3x^2$  and identify which critical points are stationary points.
10. Determine whether the statement "If the graph of  $f$  has a cusp at  $x = 1$ , then  $f$  cannot have an inflection point at  $x = 1$ " is true or false. Explain your answer.

**Turn over**

**Section B**

*All questions can be answered.  
Each question carries 6 marks.  
(Ceiling 36 marks)*

11. Find the center and radius of the circle  $x^2 + y^2 + 4x - 6y - 3 = 0$ .
12. Determine whether the statement “If  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exist, then so does  $\lim_{x \rightarrow a} [f(x) + g(x)]$ ” is true or false. Explain your answer.
13. Define  $h(2)$  in a way that extends  $h(t) = \frac{t^2 + 3t - 10}{(t - 2)}$  to be continuous at  $t = 2$ .
14. Find  $\frac{dy}{dx}$  if  $2y = x^2 + \sin y$ .
15. Find the absolute extrema of  $f(x) = \sqrt{x} + \cos x$  on  $\{0, 2\pi\}$ .
16. Find the interval on which the function  $f(x) = ax^2 + bx + c$ ,  $a \neq 0$ , is increasing and decreasing. Describe the reasoning behind your answer.
17. Find the asymptotes of the graph of  $f(x) = -\frac{8}{x^2 - 4}$ .
18. Solve  $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 4}$ .

**Section C**

*Answer any one question.  
The question carries 10 marks.*

19. (a) Suppose  $\lim_{x \rightarrow 1} f(x) = 2$  and  $\lim_{x \rightarrow 1} g(x) = 9$ . Find  $\lim_{x \rightarrow 1} \frac{f(x) - x^2 g(x)}{x - f(x)}$ .
- (b) Explain the continuity of the function  $f(x) = \frac{3x + 4}{x^2 - 4}$ .
20. (a) Suppose that  $f(-1) = 3$  and  $f'(x) = 0$  for all  $x$ . Must  $f(x) = 3$  for all  $x$ ? Give reasons for your answer.
- (b) Discuss the concavity and convexity of the curve  $y = x^2$ .

(1 × 10 = 10 marks)