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C 83073 (Pages : 3) Name..

Reg. No.......................................

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION

JUNE 2020

(CBCSS)

Physics

PHY 2C 07—STATISTICAL MECHANICS

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Section A

Answer **all** *questions.*

Each question carries a weight of 1.

1. Explain what is meant by thermodynamic limit.

2. Write a brief note on the formula $S = k \ln \Omega$.

3. What do you mean by a grand canonical ensemble ?

4. Explain the importance of the symmetry property of density matrix given as $\hat{\rho}_{mn} = \hat{\rho}_{nm}$.

5. For a canonical ensemble, derive the formula for the density matrix given below

$$\hat{\rho} = \frac{e^{-\beta\hat{H}}}{Tr\left(e^{-\beta\hat{H}}\right)}.$$

6. Explain the terms : (a) Identical particles ; and (b) Indistinguishable particles.

7. What do you mean by an ideal Fermi gas ?

8. For a Fermi-Dirac distribution the mean occupation number for single particle state is given by,

$$\langle n_\varepsilon \rangle = \frac{1}{e^{(\varepsilon - \mu)/kT} + 1}.$$

Show graphically how this function varies with temperature.

(8 × 1 = 8 weightage)

Turn over

Section B

Answer any **two** *questions.*
Each question carries a weight of 5.

9. Explain the Gibbs paradox and its resolution by deriving the Sackur-Tetrode formula.

10. Derive expressions for energy fluctuations in the case of canonical ensemble.

11. Find the expressions for grand partition function in the cases of Bose-Einstein and Fermi-Dirac distribution assuming ideal gas conditions.

12. Derive the formula for specific heat of a solid in terms of Einstein function.

(2 × 5 = 10 weightage)

Section C

Answer any **four** *questions.*
Each question carries a weight of 3.

13. Show that in the case of an ideal gas undergoing a reversible adiabatic process $PV^{5/3}$ is a constant.

14. Show that the entropy of a system in a grand canonical ensemble can be written as,

$$S = -k \sum_{r,s} P_{r,s} \ln P_{r,s},$$

where

$$P_{r,s} = \frac{\exp(-\alpha N_r - \beta E_s)}{\sum_{r,s} \exp(-\alpha N_r - \beta E_s)}.$$

15. Which of the following matrices qualify as a density matrix for a two level system? Qualify your answer in each case.

$$\rho_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \rho_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \rho_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

16. Show that for a free particle confined to a box of volume V the partition function is,

$$Q_1(\beta) = Tr\left(e^{-\beta \hat{H}}\right) = V \text{ left}\left(\frac{m}{2\pi \beta \hbar^2}\right)^{3/2}$$

where

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2.$$

17. Give a qualitative argument for treating photons and phonons as bosons.

18. State and explain equipartition theorem.

19. Given the following data, make an estimate of the Fermi energy of free electron gas in metallic sodium and express it in the units of electron volts. Effective mass of electron = 8.9×10^{-31} kg, charge $e = 1.6 \times 10^{-19}$ C, no. of conduction electrons per atom = 1, atoms per unit cell = 1, lattice constant $a = 4.29 \times 10^{-10}$ m, Planck's constant $h = 6.6 \times 10^{-34}$ Js.

(4 × 3 = 12 weightage)