D 140191 **(Pages : 3)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER (CBCSS—U.G.) DEGREE EXAMINATION
APRIL 2026

Mathematics

MTS6B10—REAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Ceiling is 25.

1. State discontinuity criterion for a function at a point.
2. Give an example of a continuous function which is not bounded.
3. Define Lipschitz function.
4. Define norm of a partition. Let I = [0, 4]. Calculate the norm of the partition $\mathcal{P} = (0, 1, 2, 4)$.
5. Prove that any step function is integrable.
6. State and prove product theorem.
7. Define null set and give an example of a null set.
8. State first form of Fundamental theorem of calculus.
9. Define uniform convergence of a sequence of functions.
10. Evaluate $\lim \dfrac{nx}{1+nx}$ for $x \in \mathbb{R}, x \geq 0$.
11. State Cauchy criterion for uniform convergence of a series of functions.
12. Evaluate $\int_{-1}^{0} \dfrac{dx}{\sqrt[3]{x}}$.

Turn over

13. Define Cauchy principal value of an improper integral.

14. Define integral test.

15. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Ceiling is 35.

16. State and prove boundedness theorem.

17. Let I be a closed bounded interval and let $f : \mathrm{I} \to \mathbb{R}$ be continuous on I. If $\in > 0$, prove that there exists a step function $s_{\in} : \mathrm{I} \to \mathbb{R}$ such that $|f(x) - s_{\in}(x)| < \in$ for all $x \in \mathrm{I}$.

18. State and prove squeeze theorem for Riemann integrals.

19. If $f : [a, b] \to \mathbb{R}$ is monotone on $[a, b]$, then prove that $f \in \mathcal{R}[a, b]$.

20. Prove that a sequence (f_n) of bounded functions on $\mathrm{A} \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_{\mathrm{A}} \to 0$.

21. Prove that all $q \in \mathbb{R}, \int_1^{\infty} x^q e^{-x}\, dx$ converges.

22. Evaluate $\int_0^{\infty} \sqrt[3]{y} e^{-y^2}\, dy$.

23. Prove that $\Gamma(2p) = \dfrac{2^{2p-1/2}}{\sqrt{2\pi}} \Gamma(p)\ \Gamma(p + 1/2)$.

Section C

Answer any **two** *questions.*

Each question carries 10 *marks.*

Maximum 20 *marks.*

24. (a) State and prove uniform continuity theorem.

    (b) Prove that $g(x) = \sqrt{x}$ is uniformly continuous on $A = [0, \infty)$.

25. Let $f : [a, b] \to \mathbb{R}$ and let $c \in (a, b)$. Prove that $f \in \mathcal{R}[a, b]$ if and only if its restriction to $[a, c]$ and $[c, b]$ are both Riemann integrable and in this case $\int_a^b f = \int_a^c f + \int_c^b f$.

26. Let (f_n) be a sequence of functions in $\mathcal{R}$ $[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f. Then prove that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \to \infty} \int_a^b f_n$.

27. State Cauchy test. Prove that $\int_0^\infty \frac{\sin x}{x}\, dx$ exists.