C 20209 **(Pages : 4)** **Name..................................**

Reg. No..............................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 11—NUMERICAL METHODS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the minimum number of iterations required in bisection method to achieve an accuracy $\in$?
2. State the condition for convergence of Newton-Raphson method.
3. Define the central difference operator.
4. Evaluate $\Delta\left(x^2 + \sin x\right)$, interval of differencing being h.
5. State Newton's backward difference interpolation formula.
6. Show that the Lagrange interpolating polynomial is unique.
7. Given $f(x) = \dfrac{1}{x^2}$, find the divided differences $[a, b]$ and $[a, b, c]$.
8. Given a set of n-values of (x, y), what is the formula for computing $\left[\dfrac{d^2y}{dx^2}\right]_{x_n}$.
9. State general formula for numerical integration.
10. What is complete pivoting ?
11. Write Runge-Kutta formula to fourth order to solve $\dfrac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.
12. Write Adams-Moulton corrector formula.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Given that the equation $x^{2.2}$ = 69 has a root between 5 and 8. Use the methods of Regula-Falsi to determine it.

Turn over

14. Prove that (i) $\delta \equiv \Delta E^{-1/2}$; (ii) $E \equiv e^{hD}$ where E is the shift operator and D is the differential operator.

15. Given $\log_{10} 100 = 2$, $\log_{10} 101 = 2.0043$, $\log_{10} 103 = 2.0128$, $\log_{10} 104 = 2.0170$, find $\log_{10} 102$.

16. The function $y = \sin x$ is tabulated below :

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \sin x$	0	0.70711	1.0

Using Lagrange's interpolation formula, find the value of $\sin\left(\frac{\pi}{6}\right)$.

17. Prove that the nth divided difference of a polynomial of nth degree are constant.

18. Given the set of tabulated points (0, 2), (1, 3), (2, 12) and (15, 3587) satisfying the function $y = f(x)$, compute $f(4)$ using Newton's divided difference formula.

19. Using Simpson's $\frac{3}{8}$-rule with $h = \frac{\pi}{6}$, evaluate the integral $\int_0^{\pi/2} \sin x \, dx$.

20. Solve the system $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16$ by the Gauss-Jordan method.

21. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU where L is a unit lower triangular matrix and U is a upper triangular matrix.

22. Find the smallest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

23. Use Picard's method to obtain $y(0.1)$ of the problem defined by $\frac{dy}{dx} = x + yx^4, y(0) = 3$.

24. Explain briefly the method of iteration to compute a real root of the equation $f(x) = 0$, stating the condition of convergence of the sequence of approximations.

25. A rod is rotating in a plane about one of its ends. The angle θ (in radians) at different times t (seconds) are given below :

t	0	0.2	0.4	0.6	0.8	1.0
θ	0.0	0.15	0.50	1.15	2.0	3.20

Find its angular acceleration when $t = 0.6$ seconds.

26. Solve the tridiagonal system of equations $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Using the secant method, find a real root of the equation $f(x) = xe^x - 1 = 0$.

28. Using bisection method find the positive root, between 0 and 1, of the equation $x = e^{-x}$ to a tolerance of 0.05 %.

29. Using Newton's forward interpolation formula, find y at $x = 8$ from the following table :

x	0	5	10	15	20	25
y	7	11	14	18	24	32

30. From the following table, find the value of $e^{1.17}$ using Gauss' forward formula :

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
e^x	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

31. Given the table of values

x	2	3	4	5
x^3	8	27	64	125

Use the method of successive approximations to find x when $x^3 = 10$.

32. Find the first and second derivatives of the function tabulated below at the point $x = 2.2$:

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

33. Use Gauss elimination to find the inverse of the matrix $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 2 & 2 \end{bmatrix}$.

34. If $\dfrac{dy}{dx} = \dfrac{1}{x^2 + y}$ with $y(4) = 4$ compute the values of $y(4.1)$ and $y(4.2)$ by Taylor's series method.

Turn over

35. A curve is given by the points of the table given below :

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	23	19	14	11	12.5	16	19	20	20

Apply Simpson's rule to find the area bounded by the curve, the x-axis and the extreme ordinates.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 marks.

36. Evaluate $\int_0^1 \frac{dx}{1+x}$ using :

(a) Trapezoidal rule taking $h = 0.25$.

(b) Simpson's $\frac{1}{3}$-rule taking $h = 0.125$.

37. Solve the system $10x + 2y + z = 9; 2x + 20y - 2z = -44; -2x + 3y + 10z = 22$ using both Jacobi and Gauss-Seidel method.

38. (a) Use Runge-Kutta fourth order formula to find $y(0.2)$ and $y(0.4)$ given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1.$

(b) Solve the initial value problem $\frac{dy}{dx} = 1 + y^2, y(0) = 0$ with $h = 0.2$ on the interval [0, 0.6] using Milne's method.

(2 × 13 = 26 marks)