C 20208 **(Pages : 3)** Name.....................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2014 to 2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. A complex function $f(z)$ is analytic at a point $z = z_0$ if ————.
2. An analytic function with constant argument is ————.
3. Give an example of a complex function which is Differentiable at a point but not analytic at that point.
4. Find the simple poles, if any for the function $f(z)=\dfrac{(z+2)^2}{z^5\left(x^4-1\right)}$.
5. Write the polar form of Cauchy-Riemann equations.
6. Define residue of a complex valued function.
7. Fill in the blanks : The real part of sinh $(2z)$ is ————.
8. Fill in the blanks : $f(z) = e^z$ is periodic with period = ————.
9. A point $z = z_0$ is a singular point of a complex function $w = f(z)$ if ————.
10. Fill in the blanks : $\text{Res}_{z=\pi/2} \tan z =$ ————.
11. The solution of the equation $e^z = -3$ is ————.
12. The principal value of i^i is ————.

$(12 \times 1 = 12$ marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Show that $f(z) = \sin z$ is analytic for all z.
14. Find the principal value of $(1-i)^{1+i}$.
15. Show that $\tanh^{-1}(z)=\dfrac{1}{2}\log\dfrac{1+z}{1-z}$.
16. Show that the zeros of an analytic function are isolated.

Turn over

17. Determine and classify the singular points of $f(z)=\frac{(z+2)^2}{z^5\left(z^4-1\right)}$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Discuss the nature of singularities if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solution of $e^z = 2$.

22. Find the residue of $f(z) = \cot(z)$ at its poles.

23. Evaluate $\oint_C \frac{\sin \pi z}{\left(z^6\right)} dz$ around $C = |z| = 1$.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Evaluate $\oint_{|z|=2} \bar{z} dz$.

26. Illustrate entire function by an example.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \frac{1}{(z-1)(z-2)}$ around the simple closed curve $C = |z| = 4$.

28. Determine the nature of the singularities of the function $f(z) = \sec(1/z)$.

29. Expand $f(z)=\frac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0<|z+1|<2$.

30. If $f(z)=u(x,y+iv(x,y))$ is analytic in a domain D, then prove that its component functions are harmonic in D.

31. Find the analytic function $f(z)$ is terms of z, if $u(x,y)=\mathrm{Re}(f(z))=e^x(x\cos y-y\sin y)$.

32. Show that the function $f(z)=\sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. State and prove Morera's theorem.

34. Show that the derived series has the same radius of convergence as the original series.

35. Evaluate $\oint_{|z-2|=2} \frac{z^3}{(z-1)^4(z-2)(z-3)} dz$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 marks.

36. (a) State and prove Cauchy's integral formula.

(b) Prove or disprove : $|\cos(z)| \leq 1$ for all complex numbers z. Justify your claim.

37. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{a + \cos\theta} d\theta$.

(b) Evaluate $\int_0^{\infty} \dfrac{1}{x^4 + a^4} dx, a > 0$.

(2 × 13 = 26 marks)