Q.P Code :D142007	Total Pages : 3	Name 760648
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SECOND SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2026

MATHEMATICS

MAT2MN103
Analysis and Some Counting Principles

2024 Admission Onwards

Maximum Time :2 Hours	**Maximum Marks :70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

1	With proper arguments, show that the sequence $\left\{\dfrac{1}{n^4}\right\}$ is convergent.
2	True or False: "The series $\displaystyle\sum_{n=1}^{\infty}\frac{n}{(n+1)}$ is convergent.". Give reasons
3	Let $z = x + iy$. Express the following quantity in terms of x and y $\mathrm{Im}(2z^2 + 4\bar{z} - 4(1/z))$
4	Determine which of the two complex numbers $10 + 8i$ and $11 - 6i$ is closest to the origin. Which is closest to $1 + i$?
5	Find the range of each of the complex function $f(z) = \mathrm{Im}(z)$ defined on the closed disk $\lvert z\rvert \leq 2$
6	Find $\displaystyle\lim_{z\to -i}\frac{z^4-1}{z+i}$
7	Using the Definition of Derivative find the derivative of $\dfrac{1}{z}$
8	Find the number of distinguishable permutations of the letters in REQUIREMENTS.
9	In how many ways can five balls be chosen so that at most three are black?
10	Show that if seven colors are used to paint 50 bicycles, at least eight bicycles will be the same color.

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Section B

All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks))

11	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ by integral test
12	Use the Limit Comparison Test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \sin \frac{1}{n}$
13	Find $8^{1/3}$
14	Sketch the set S of points in the complex plane satisfying the inequality $\mathrm{Im}(z) < \mathrm{Re}(z)$. Determine whether the set is (a) open, (b) closed, (c) a domain, (d) bounded, or (e) connected.
15	Prove that "If f is differentiable at a point z_0 in a domain D, then f is continuous at z_0.
16	Suppose the function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D. Prove that if $\lvert f(z)\rvert$ is constant in D, then so is $f(z)$.
17	Five fair coins are tossed and the results are recorded. (a) How many different sequences of heads and tails are possible? (b) How many of the sequences in part (a) have exactly one head recorded? (c) How many of the sequences in part (a) have exactly three heads recorded?
18	Ten people volunteer for a three-person committee. Every possible committee of three that can be formed from these ten names is written on a slip of paper, one slip for each possible committee, and the slips are put in ten hats. Show that at least one hat contains 12 or more slips of paper.

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))

19	Prove that:"If the series $\sum_{n=1}^{\infty} \lvert a_n \rvert$ an converges, then the series $\sum_{n=1}^{\infty} a_n$ an also converges.". Is the converse true? Why?
20	Consider the function $f(z) = \begin{cases} 0 & \text{when } z = 0 \\ \dfrac{z}{\lvert z \rvert} & \text{when } z \neq 0 \end{cases}$ (a) Express f in the form $f(z) = u(x, y) + iv(x, y)$ (b) Show that f is not differentiable at the origin. (c) Check whether Cauchy-Riemann equations are satisfied at the origin.