D 140197 **(Pages : 2)** Name..

Reg. No.....................................

SIXTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2026

Mathematics

MTS 6B 14 (E01)—GRAPH THEORY

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Part A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 20.

1. Define graph isomorphism.
2. Prove that the number of edges of the complete graph of n vertices, K_n is $\frac{n(n-1)}{2}$.
3. Define k-regular graph. Give an example of a 3-regular graph with 10 vertices.
4. What is the resulting graph of K_9, when one vertex is deleted from it ? Explain.
5. Draw two non-isomorphic spanning trees of K_5.
6. What you mean by internally disjoint paths ? Explain using an example.
7. Find the vertex connectivity of the complete graph K_5 ? Justify.
8. Distinguish between trees and Forests, explain with example.
9. Define Eulerian graphs. Give a characterization of Eulerian graphs.
10. State Dirac sufficient condition for a graph to be Hamiltonian.
11. Is the complete graph K_8 Euler ? Justify your answer ?
12. State Euler's theorem for planar graphs. Verify the theorem for $K_{2,3}$.

Part B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 30.

13. Prove the it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.

14. Given any *two* vertices u and v of a graph G, prove that every $u - v$ walk contains a $u - v$ path.

15. Let G be a graph without any loops. If for every pair of distinct vertices u and v of G there is precisely one path from u to v, then prove that G is a tree.

16. Let G be a connected graph. Then prove that G is a tree if and only if for every edge e of G the subgraph $G - e$ has two components.

17. Let v be a vertex of the connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G, both different from v, such that v is on every $u - w$ path in G.

18. Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq n$. Let $G + uv$ denote the supergraph of G obtained by joining u and v by an edge. Then prove that G is Hamiltonian if and only if $G + uv$ is Hamiltonian.

19. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.

Part C

Answer any **one** *question.*
The question carries 10 marks.

20. For any nonempty graph G with at least two vertices, prove that G is bipartite if and only if it has no odd cycles.

21. If T is a tree with n vertices then prove that it has precisely $n - 1$ edges.

(1 × 10 = 10 marks)