D 70320 (Pages : 4) Name..................................

Reg. No..................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Evaluate $\lim_{(x, y) \to (1, 3)} \frac{x+1}{4-y}$.

2. Find the domain and range of $z = \sqrt{1 - x^2 - y^2}$.

3. Find the gradient of $\phi(x, y, z) = x^2 + y^2 + z^2$.

4. Compute the divergence of $\vec{f} = xy\,\vec{i} + yz\,\vec{j} + xz\,\vec{k}$.

5. Define directional derivative of a function.

6. What do you mean by a conservative vector field ?

7. Give a very brief discription of linearization of a function of two variables.

8. Find du if $u = e^{x^2 + y^2 + z^2}$.

9. Fill in the blanks : If $\vec{f}$ and $\vec{g}$ are differentiable vector point functions, then

   $\nabla \cdot \left(\vec{f} \times \vec{g}\right) =$ ....

10. State the tangential form of Green's theorem in the plane.

Turn over

23. Test whether the vector $\vec{f} = \left(e^x \cos y + yz\right)\vec{i} + \left(xz - e^x \sin y\right)\vec{j} + (xy + z)\ \vec{k}$ is conservative or not.

24. If the sides and angles in a traingle vary in such a way that its circum-radius R remains a constant, then show that $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0.$

25. Verify whether the differential $ydx + xdy + 4dz$ is exact or not.

26. Show that $\vec{f} \times \vec{g}$ is solenoidal if $\vec{f}$ and $\vec{g}$ are irrotational.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*

Each question carries 7 marks.

27. Evaluate $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x}\, dydx.$

28. If $\vec{f}$ is a differentiable vector function of t, differentiable at least 3 times, prove that $\frac{d}{dt}\left[\vec{f}, \vec{f}', \vec{f}''\right] = \left[\vec{f}, \vec{f}', \vec{f}'''\right].$

29. Find the work done by the force field $\vec{f} = z\,\vec{i} + x\vec{j} + y\,\vec{k}$ along the boundary of the curve $C : \vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + 3t\,\vec{k}$ where $0 \le t \le 2\pi$.

30. Test the continuity of $f(x, y)$ defined by

$$f(x, y) = \frac{xy}{x^2 + y^2}, (x, y) \ne (0, 0) \text{ and } f(x, y) = 0,\ (x, y) = (0, 0).$$

31. Find the equation to the tangent plane and normal line to the surface $f(x, y, z) = x^2 + y^2 + z^2 - 9 = 0$ at the point (1, 2, 4).

32. Evaluate the area enclosed by the Lemniscate $r^2 = 4 \cos 2\theta$ using double integrals.

33. Find the Local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

34. Evaluate the volume of the region bounded by $x^2 + y^2 = 4$, $y + z = 3$, $z = 0$.

35. Show that $\vec{f} = y \sin z\, \vec{i} + x \sin z\, \vec{j} + xy \cos z\, \vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy\, \vec{i} + yz\, \vec{j} + xz\, \vec{k}$ through the surface of the cube cut from the first octant by the planes $x = y = z = 1$.

(b) If S is a closed surface enclosing a volume V, then prove that $\iint_S \vec{r}.n dS = 3V$.

37. (a) Evaluate the surface integral $\iint_S \vec{f}.n dS$ where $\vec{f} = y\, \vec{i} + x\, \vec{j} + z^2\, \vec{k}$ over the cylindrical surface S given by $x^2 + y^2 = a^2$, $z = 0$, $z = h$.

(b) Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -2, 2)$.

38. (a) Find the value of $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz$.

(b) In what direction from the point $(2, 1, -1)$ the directional derivative of $\phi(x, y, z) = x^2 yz^3$ is maximum and find the magnitude of this maximum.

(2 × 13 = 26 marks)