

D 12646

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Name.....

Reg. No.....

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION  
NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2021 Admissions)

Time : Two Hours

Maximum : 60 Marks

## Section A

*Answer at least eight questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 24.*

1. Calculate the slope of the tangent line to the graph of  $f(x) = x^2 + 1$  when  $x = -1$ .
2. Find  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$ .
3. Find the derivative of  $y = \sqrt{x}$  for  $x > 0$ .
4. Find  $\frac{d}{dx} [\cos(\sqrt{1 + \cos x})]$ .
5. Find the linearization of  $f(x) = \cos x$  at  $x = \pi/2$ .
6. Show that there is a number  $c$  such that  $c^3 - c^2 = 10$ .
7. Find  $\lim_{t \rightarrow 0} \cos\left(\frac{x}{\sqrt{19 - 3 \sec 2t}}\right)$ .
8. Suppose that  $f$  is differentiable on the whole real line and that  $f'(x)$  is constant. Prove that  $f$  is linear.

Turn over

9. Find the critical points of  $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$ .
10. Find the inflection points of  $f(x) = x^2 + (1/x)$ .
11. Using limits of Riemann sums, establish the equation  $\int_a^b c \, dx = c(b - a)$ , where  $c$  is a constant.
12. Find  $\int_0^2 \left( \frac{t^2}{4} - 7t + 5 \right) dt$ .

(8 × 3 = 24 marks)

### Section B

Answer at least **five** questions.

Each question carries 5 marks.

All questions can be attended.

Overall Ceiling 25.

13. Find  $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$ .
14. Show that the line  $y = mx + b$  is its own tangent at any point  $(x, mx + b)$  on the line.
15. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 1 ft/s. How fast is the area of the spill increasing when the radius of the spill is 20 ft?
16. Use implicit differentiation to find  $d^2y/dx^2$  if  $5x^3 - 7y^2 = 10$ .
17. Find the maximum and minimum points and values for the function  $f(x) = (x^2 - 8x + 12)^4$  on the interval  $[-10, 10]$ .
18. Use l'Hôpital's Rule to find  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$ .

19. Find the area of the region between the  $x$ -axis and the graph of  $f(x) = x^3 - x^2 - 2x$ ,  $-1 \leq x \leq 2$ .

(5 × 5 = 25 marks)

### Section C

*Answer any one question.  
The question carries 11 marks.*

20. (a) Find the area of the region in the first quadrant that is bounded above by  $y = \sqrt{x}$  and below by the  $x$ -axis and the line  $y = x - 2$ .

(b) Find  $\frac{dy}{dx}$  if  $y = \int_1^{x^2} \cos t \, dt$ .

21. (a) Find the absolute extrema of  $h(x) = x^{2/3}$  on  $[-2, 3]$ .

- (b) Find the volume of the solid generated by the revolution about the  $x$ -axis of the loop of the

curve  $y^2 = x^2 \frac{3a - x}{a + x}$ .

(c) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$ .

(1 × 11 = 11 marks)