D 52331 (Pages : 3) Name..............................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2018

(CUCSS–PG)

Mathematics

MT 3C 15—PDE AND INTEGRAL EQUATIONS

(2016 Syllabus Year)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Determine the partial differential equation satisfied by the surfaces of the form F $(u, v) = 0$, where $u = u(x, y, z)$ and $v = v(x, y, z)$ are known functions of x, y and z and F is an arbitrary function of u and v.

2. If $(x-a)^2+(y-b)^2+z^2=1$ is a complete integral of $z^2\left(1+p^2+q^2\right)=1$, then find the singular integrals.

3. Determine the characteristic curves for the initial value problem $zz_x+z_y=1$ with the initial conditions $x = s, y = s,\ z=\frac{1}{2}s, 0 \le s \le 1$.

4. Show that the equation $p^2+q^2=1$ and $\left(p^2+q^2\right)x=pz$ are compatible.

5. Determine the Monge cone with vertex (0, 0, 0) for the equation $p^2+q^2=1$.

6. Find the Riemann function for the wave equation in Canonical form $u_{\xi\eta}=0$.

7. State maximum and minimum principles for harmonic functions.

8. Show that the solution of the Neumann problem is unique upto the addition of a constant.

9. Show that the solution to the Dirichlet problem is stable.

10. State the Dirichlet problem for a rectangle.

11. Differentiate between Fredholm and Voltera integral equations.

Turn over

12. Determine $p(x)$ and $q(x)$ in such a way that the equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ is equivalent to the equation $\frac{d}{dx}\left(p\frac{dy}{dx}\right)+qy=0$.

13. Determine the iterated Kernel $k_2(x,\xi)$ associated with $k_2(x,\xi)=|x-\xi|$ in the interval (0, 1).

14. Show that the Kernel $k(x,\xi)=1+\xi+3x\xi$ has a double characteristic number associated with (–1, 1), with only one characteristic function.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Find the general integral of $(y+1)p+(x+1)q=z$.

16. Show that the Pfaffian differential equation $yz\,dx+\left(x^2y-zx\right)dy+\left(x^2z-xy\right)dz=0$ is exact and find the corresponding integral.

17. Solve by Jacobi's method :

$$u_x\,x^2-u_y{}^2-au_z^2=0.$$

18. Find the characteristic strips of the equation $xp+yq=pq$ where the initial curve is $c:z=\frac{x}{2},y=0.$

19. Obtain the d' Alembert's solution which describes the vibrations of an infinite string.

20. Solve the Dirichlet problem for the upper half plane.

21. Solve the Neumann problem for a circle.

22. Determine the Green's function for the Bessel operator of order $n\neq 0, Z_y=\frac{d}{dx}\left(x\frac{dy}{dx}\right)-\frac{n^2}{x}y,$ relevant to the end conditions $y(0)=y(1)=0$.

23. If the Kernel $k(x,\xi)$ of the integral equation $y(x)=\lambda\int_a^b k(x,\xi)\,y(\xi)\,d\xi$ is symmetric, then show that the characteristic functions of the equation corresponding to distinct characteristic numbers are orthogonal over the interval (a,b).

24. Solve the equation by iterative method :

$$y(x)=\lambda\int_0^1(x+\xi)\,y(\xi)\,d\xi+1.$$

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. Find the complete integral of the equation $p^2x + qy = z$ and derive the equation of the integral surface containing the line $y = 1$, $x + z = 0$.

26. Reduce the equation $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$, to a Canonical form and solve it.

27. Solve the heat conduction problem in an infinite rod.

28. Consider the equation $y(x) = F(x) + \lambda \int_0^{2\pi} \cos(x+\xi)\, y(\xi)\, d\xi$.

    (a) Determine the characteristic values of λ and the corresponding characteristic functions.

    (b) Express the solution in the form $y(x) = F(x) + \lambda \int_0^{2\pi} \Gamma(x, \xi ; \lambda) F(\xi)\, d\xi$ when λ is not characteristic and obtain the general solution (when it exists) if $F(x) = \sin x$, considering all possible cases.

(2 × 4 = 8 weightage)