D 72969 (Pages : 3) Name..

Reg. No..

FIRST SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION DECEMBER 2019

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA—I

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*

Each question has weightage 1.

1. Verify whether $\phi(x, y) = (x, y) + 1$ is an isometry of the plane.
2. Find the order of $(1, 2)$ in the group $\mathbb{Z}_3 \times \mathbb{Z}_4$.
3. Describe all abelian groups of order 36 upto isomorphism.
4. Find all homomorphisms from $\mathbb{Z}_4 \times \mathbb{Z}_{12}$.
5. Let G be a group of order 20. Find the number of 5-Sylow subgroups of G.
6. Give a presentation of the Klein 4 group using two generators.
7. Verify whether $(x-2)$ is a factor of $x^3 - 3x^2 + 3x - 2$ in $\mathbb{Q}[x]$.
8. Let $\phi : \mathbb{Z} \to \mathbb{Z}$ be a map defined by $x \mapsto 2x$. Verify whether ϕ is a ring homomorphism.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions choosing two from each unit.*

Each question has weightage 2.

Unit 1

9. Describe an isomorphism ϕ from $\mathbb{Z}_4 \times \mathbb{Z}_5$ to $\mathbb{Z}_{20}$. Verify that ϕ is an isomorphism.

Turn over

10. Let H be a normal subgroup of a group G and $a, b \in G$. Show that if $x \in aH$ and $y \in bH$ then $xy \in (ab)H$.

11. Let M be a maximal normal subgroup of a group G. Show that G/M is simple.

UNIT 2

12. Give a composition series for the symmetric group S_3.

13. Let G be a group of order 45. Show that G has a normal subgroup of order 5.

14. Find all elements conjugate to $(1\ 2\ 3)$ in S_4.

UNIT 3

15. List all elements in the group algebra FG where F is the field Z_2 and G is the cyclic group of order 2. Give the multiplication table for the product in FG.

16. Let $\phi_\pi : \mathbb{Q}[x] \to \mathbb{Q}$ be the evaluation homomorphism with $\phi_\pi(x) = \pi$. Find the kernel of ϕ_π.

17. Show that $N = \{f \in \mathbb{R}[x] : f(1) = 0\}$ is a maximal ideal in $\mathbb{R}[x]$.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Let H be a subgroup of a group G. Show that the following are equivalent.

(i) $ghg^{-1} \in H$ for all $g \in G$ and $h \in H$.

(ii) $gHg^{-1} = H$ for all $g \in G$.

(iii) $gH = Hg$ for all $g \in G$.

(b) Show that every subgroup of an abelian group is a normal subgroup.

19. Let X be G-set and $g \in G$. Show that :

(a) $\sigma_g : X \to X$ defined by $x \mapsto gx$ is one to one and onto.

(b) For $x \in X$ let $G_x = \{g \in G : gx = x\}$. Then G_x is a subgroup of G.

20. (a) Let F be a free group on a set A and G be any group. Let $f : A \to G$ be a map. Show that there is a homomorphism $\phi : F \to G$ such that $\phi(a) = f(a)$ for all $a \in A$.

(b) Show that every group is a homomorphic image of a free group.

21. (a) Let F be a field and $f(x) \in F[x]$ be of degree 2 or 3. Show that $f(x)$ is irreducible if and only if $f(x)$ has no zero in F.

(b) State Eisenstein criterion for irreducibilty of a polynomial.

(c) Show that the polynomial $x^5 + 6x^3 + 4x + 10$ is irreducible in $Q[x]$.

(2 × 5 = 10 weightage)