D 123853 (Pages : 12) Name.....

Reg. No.....

CALICUT UNIVERSITY CENTRALIZED ENTRANCE TEST (CU-CET)
APRIL 2025

M.Sc. MATHEMATICS

Time : Two Hours Maximum : 200 Marks

Symbols

$\mathbb{R}$: The set of all real numbers

$\mathbb{C}$: The set of complex numbers

$\mathbb{Z}$: The set of all Integres

$\mathbb{Q}$: The set of all rational numbers

det (A) : determinant of A.

Each question carries 4 marks.

1 mark will be deducted for each wrong answer.

1. Find the distance between the points $P(1,-4)$ and $P(4,-8)$ on the two dimensional plane :

(A) $2\sqrt{5}$.

(B) 5.

(C) $\sqrt{5}$.

(D) $5\sqrt{5}$.

2. Which one among the following is a cube root of unity ?

(A) $\frac{1+i\sqrt{3}}{2}$.

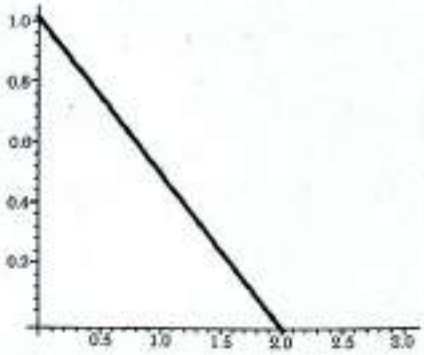
(B) $1-i\sqrt{3}$.

(C) $\frac{-1+i\sqrt{3}}{2}$.

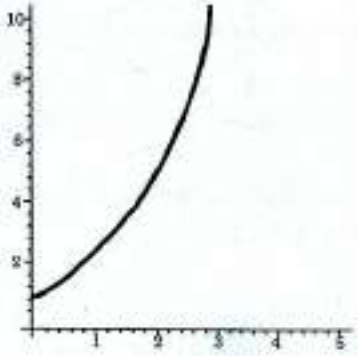
(D) $-1+i\frac{\sqrt{3}}{2}$.

3. Identify the solution of the initial value problem $\frac{dy}{dx}=x-y,\ y(0)=1$ on the interval $[0,3]$.

(A)



(B)



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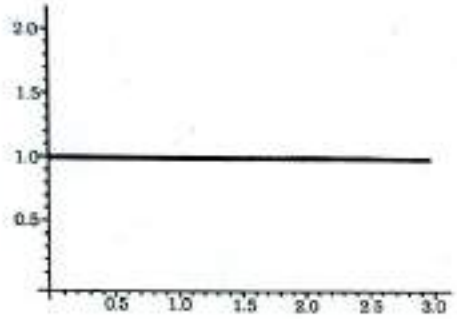

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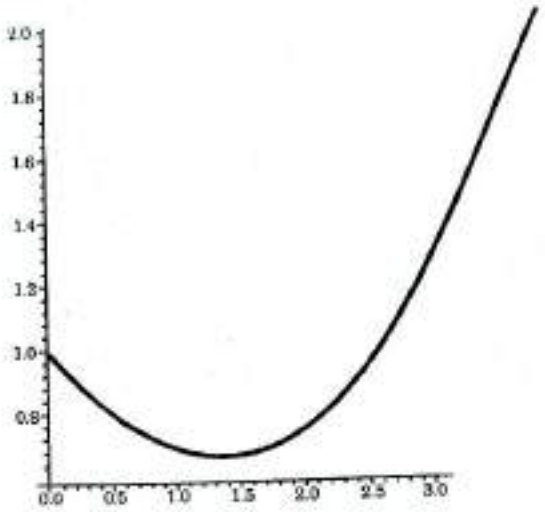


4. What is the radius of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$?

(A) 4. (B) 3.

(C) 6. (D) 13.

5. Consider $f(x) = \dfrac{\sqrt{2+x} - \sqrt{2}}{x}$. What will be $\lim\limits_{x \to 0} f(x)$?

(A) $\dfrac{1}{2}$. (B) $\sqrt{2}$.

(C) $\dfrac{1}{\sqrt{2}}$. (D) $\dfrac{1}{2\sqrt{2}}$.

6. Which of the following statement is correct ?

(A) There exists a number exactly 1 less than its cube in the interval $[0,1]$.

(B) There exists a number exactly 1 less than its cube in the interval $[-1,0]$.

(C) There exists a number exactly 1 less than its cube in the interval $[1,2]$.

(D) There does not exists a number exactly 1 less than its cube on the real line.

7. What is the rate of change in area of a circle with respect to the diameter when the radius is 5 m ?

(A) $-5\pi m^2/m$.

(B) $10\pi m^2/m$.

(C) $-10\pi m^2/m$.

(D) $5\pi m^2/m$.

8. What is the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point $P(3, -4)$?

(A) $\frac{3}{4}$.

(B) $-\frac{3}{4}$.

(C) $\frac{4}{3}$.

(D) $-\frac{4}{3}$.

9. What is the largest area of a rectangle possible if inscribed in a semicircle of radius 2 units ?

(A) $2\sqrt{2}$ square units.

(B) 4 square units.

(C) $8\sqrt{2}$ square units.

(D) 16 square units.

10. What is the linearization of the function $f(x) = \sqrt{1+x}$ locally at $x = 0$?

(A) $1 + \frac{x}{4}$.

(B) $\frac{1+x}{2}$.

(C) $1 + \frac{x}{2}$.

(D) $1 - \frac{x}{4}$.

11. Which one among the following curves has slope of its tangent at the point (x, y) is $3x^2$ and passes through the point $P(1, -1)$?

(A) $x^3 + 2$.

(B) $x^3 - 4$.

(C) $x^3 - 2$.

(D) $x^3 + 4$.

12. What is the value of the definite integral $\int_{-1}^{1} 3x^2 \sqrt{x^3+1}\, dx$?

(A) $\frac{2\sqrt{2}}{3}$.

(B) $\frac{3\sqrt{2}}{4}$.

(C) $\frac{4\sqrt{2}}{3}$.

(D) $\frac{8\sqrt{2}}{3}$.

13. Choose correct statement from the following :

(A) If $\det(A)=0$, then the equation $Ax=y$ has at least one solution for every $y \in \mathbb{R}^n$.

(B) If the equation $Ax=y$ has a solution for every $y \in \mathbb{R}^n$, then $\det(A) \neq 0$.

(C) If A and B are invertible $n \times n$ matrices, then $\det(A+B) \neq 0$.

(D) If $A^2 = A$, then -1 is an eigenvalue of A.

14. What is the area of the region enlosed by the parabola $y = 2 - x^2$ and the line $y = -x$?

(A) $\frac{7}{2}$.

(B) $\frac{9}{2}$.

(C) $\frac{11}{2}$.

(D) $\frac{13}{3}$.

15. What is the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x=0$ to $x=2$?

(A) $\frac{1}{27}\left(\sqrt{10}-1\right)$.

(B) $\frac{2}{27}\left(\sqrt{11}-10\right)$.

(C) $\frac{2}{27}\left(10\sqrt{10}-1\right)$.

(D) $\frac{2}{27}\left(10\sqrt{11}-1\right)$.

16. Choose correct statement from the following :

(A) $S(z)=\overline{z}$ satisfy Cauchy-Riemann Equations.

(B) $S(z)=\overline{z}$ is an analytic function.

(C) $S(z)=\overline{z}$ is a harmonic function.

(D) $S(z)=\overline{z}$ is not an analytic function.

17. Let $\phi : \mathcal{R} \to \mathcal{S}$ is a ring homomorphism. Which one of the following statements is correct ?

(A) ϕ is injective if and only if ker ϕ is empty.

(B) ker ϕ is not an ideal of $\mathcal{R}$.

(C) ker ϕ is an ideal of $\mathcal{R}$.

(D) The image $\phi(\mathcal{R})$ need not be a subring of $\mathcal{S}$.

18. If $f(z)$ is an entire function with $|f(z)|<2024$ for all z, then which of the following statement is correct ?

(A) $f(z)=0$ for all z.

(B) $f(z)=2023$ for all z.

(C) $f(z)$ is a constant function

(D) The equation $f(z)=2$ has only one solution.

19. The function $f(z)=\frac{\sin z}{z}$ has :

(A) A simple pole at 0.

(B) A removable singularity at 0.

(C) An essential singularity at 0.

(D) None of the above.

20. Choose the correct value of $\int x^2 e^x \, dx$.

(A) $x^2e^x + 2e^x + C.$

(B) $x^2e^x - 2xe^x + C.$

(C) $\left(x^2 - 2x + 2\right)e^x + C.$

(D) $\left(x^2 + 2x - 2\right)e^x + C.$

21. What is the sum of the infinite series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots.$?

(A) $\frac{1}{11}.$

(B) $\frac{1}{6}.$

(C) $\frac{1}{17}.$

(D) $\frac{1}{91}.$

22. $3x^2 - 6xy + 3y^2 + 2x - 7 = 0$ represents :

(A) An ellipse.

(B) A circle.

(C) A parabola.

(D) A hyperbola.

23. What is the angle between the vectors $\hat{i} - 2\hat{j} - 2\hat{k}$ and $6\hat{i} + 3\hat{j} + 2\hat{k}$?

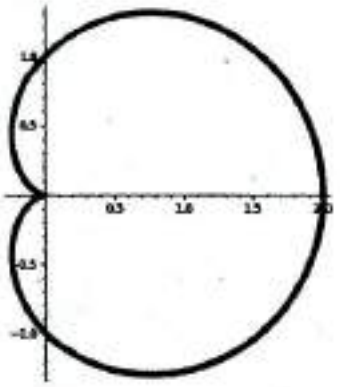
(A) $\cos^{-1}\left(\frac{4}{21}\right).$

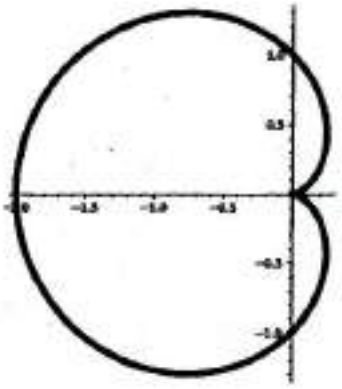
(B) $\cos^{-1}\left(\frac{4}{23}\right).$

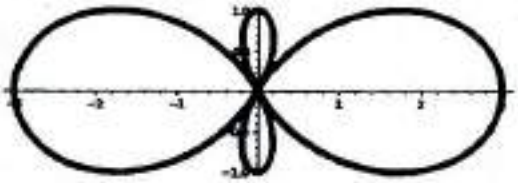
(C) $\cos^{-1}\left(-\frac{4}{21}\right).$

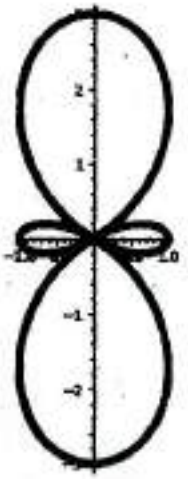
(D) $\cos^{-1}\left(-\frac{4}{23}\right).$

24. Which one among the following figure represents the function $r = 1 + \cos(\theta), 0 \leq \theta \leq 2\pi$?

(A) 

(B) 

(C) 

(D) 

25. What is the area of the triangle with vertices $P(1,-1,0), Q(2,1,-1)$ and $R(-1,1,2)$?

(A) $6\sqrt{2}$.

(B) $4\sqrt{2}$.

(C) $6\sqrt{3}$.

(D) $3\sqrt{2}$.

26. Which one among the following is the linearization of the function $f(x,y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point $(3,2)$?

(A) $4x - y - 2$.

(B) $2y - 2x + 2$.

(C) $4y + 2x - 2$.

(D) $3x + 4y - 2$.

27. What is the area of the region enclosed by the parabola $y = x^2$ and the line $= x + 2$.

(A) $\frac{7}{2}$.

(B) $\frac{9}{2}$.

(C) $\frac{11}{2}$.

(D) $\frac{11}{3}$.

28. What is the divergence of the vector field $\vec{f}(x, y) = \left(x^2 - y\right)\hat{i} + \left(xy - y^2\right)\hat{j}$?

(A) $2x + 3y$.

(B) $4x - 3y$.

(C) $3x - 2y$.

(D) $7x + 2y$.

29. What is the curl of the vectorfield $\vec{f}(x, y, z) = \left(x^2 - y\right)\hat{i} + 4z\hat{j} + x^2\hat{k}$?

(A) $3\hat{i} + 4x\hat{j} + \hat{k}$.

(B) $-4\hat{i} - 2x\hat{j} + \hat{k}$.

(C) $5x\hat{i} - 2y\hat{j} - z\hat{k}$.

(D) $7yz\hat{i} + 5xz\hat{j} + 6z\hat{k}$.

30. Suppose F_1 and F_2 are two differentiable vectorfields. Which one among the following is incorrect?

(A) $\nabla \cdot (F_1 \times F_2) = F_2 \cdot \nabla \times F_1 + F_1 \cdot \nabla \times F_2$.

(B) $\nabla \cdot (aF_1 + bF_2) = a\nabla \cdot F_1 + b\nabla \cdot F_2$.

(C) $\nabla \times (aF_1 + bF_2) = a\nabla \times F_1 + b\nabla \times F_2$.

(D) $\nabla \cdot (F_1 \times F_2) = F_2 \cdot \nabla \times F_1 - F_1 \cdot \nabla \times F_2$.

31. What is the determinat of the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$?

(A) 35.

(B) 36.

(C) 37.

(D) 38.

32. Which of the following statements about the group G of order 12 is correct ?

(A) G has no proper subgroups.

(B) G has an element of order 2.

(C) G has only one proper subgroup.

(D) G is an Abelian group.

33. Consider the mapoing $\phi : \mathbb{C}^* \to \mathbb{C}^*$ given by $\phi(x) = x^4$. Which one among the following is the kernel of this homomorphism ?

(A) $\ker \phi = \{0\}$.

(B) $\ker \phi = \{1, i\}$.

(C) $\ker \phi = \{1, -1, i, -i\}$.

(D) $\ker \phi = \{1, -1, \sqrt{3}i\}$.

34. What is the order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 4 & 7 & 1 & 8 & 3 & 6 \end{pmatrix}$?

(A) 6.

(B) 5.

(C) 4.

(D) 8.

35. How many numbers that are less than or equal to 670 and are relatively prime ?

(A) 245.

(B) 260.

(C) 264.

(D) 356.

36. Choose the solution of the system of equations $3x + y + z = 2, 6x + y + 2z = 1, 2x + 8y + 9z = -3$:

(A) $x = \frac{18}{25}, y = 33, z = \frac{19}{25}$.

(B) $x = \frac{18}{25}, y = 3, z = \frac{-79}{25}$.

(C) $x = \frac{81}{25}, y = 13, z = \frac{-19}{25}$.

(D) $x = \frac{81}{25}, y = 3, z = \frac{19}{25}$.

37. What is the rank of the given matrix $A = \begin{pmatrix} 1 & 6 & -5 \\ 4 & -6 & 7 \\ 1 & 4 & 2 \\ 3 & -2 & 4 \end{pmatrix}$?

(A) 1.

(B) 2.

(C) 3.

(D) 4.

38. Which of following is the characteristic polynomial for the given matrix $A=\begin{pmatrix}1 & 2 & -6\\ 8 & 6 & -7\\ 3 & -2 & 4\end{pmatrix}$?

(A) $x^2+11x^2-22x+108.$ (B) $x^2-11x^2+22x-108.$

(C) $2x^3-11x^2-22x+108.$ (D) $-2x^3+11x^2-22x-108.$

39. What are the characteristic values of the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 4 & 3 & 0\\ -6 & 3 & 2\end{bmatrix}$?

(A) $-2, 2, 3.$ (B) $3, 2, 1.$

(C) $0, 1, 3.$ (D) $4, -2, 6.$

40. What is the corrrect value of $(1+i)^{43}$?

(A) $2^{21}-21i.$ (B) $2^{-21}+2^{21}i.$

(C) $-2^{21}+2^{21}i.$ (D) $21+2^{21}i.$

41. Which of the following statements is correct?

(A) Pointwise convergence of a sequence of functions preserve boundedness.

(B) Pointwise convergence of a sequence of functions preserve continuity.

(C) Uniform convergence of a sequence of functions doesn't preserve continuity.

(D) Sequence of functions $f_n:\mathbb{R}\to\mathbb{R}$ defined by $f_n(x)=\frac{x}{1+nx^2}$ unifcmnly converges to 0.

42. What is the Laplace transform of the function $f(t)=e^{3t}+\cos(3t)-e^{3t}\cos(6t)$?

(A) $-\frac{s-3}{s^2-6s+45}+\frac{s}{s^2+36}+\frac{1}{s-3}.$ (B) $\frac{s-3}{s^2-6s+45}-\frac{s}{s^2+36}+\frac{1}{s+3}.$

(C) $-\frac{s+3}{s^2-6s+45}+\frac{s}{s^2-36}+\frac{1}{s-3}.$ (D) $-\frac{s-3}{s^2+6s+45}-\frac{s}{s^2+36}-\frac{1}{s-3}.$

43. Which of the following statements is incorrect ?

(A) If f is continuous on $[a, b]$, then f is Riemann integrable.

(B) All monotone functions are Riemann integrable.

(C) The function $f(x) = \begin{cases} 0 & \text{if } 0 < x \leq 1, \\ 1 & \text{if } x = 0 \end{cases}$ is Riemann integrable.

(D) The function $f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap \mathbb{Q} \\ 0 & \text{if } x \in [0,1] \setminus \mathbb{Q} \end{cases}$ is Riemann integrable.

44. A basket contains 30 blue balls and 70 pink balls. What is the probability of getting exactly 10 blue balls in a sample of size 20 if the sampling is done with replacement ?

(A) 0.0418. (B) 0.0308.

(C) 0.0234. (D) 0.2234.z

45. What is the average value of the function $f(x) = 36 - 12x$ over the interval $[4, 10]$?

(A) -44. (B) -48.

(C) 44. (D) 48.

46. Consider $F(x) = \int_x^{2x} t^3 \, dt$. What is $F'(x)$?

(A) $-12x^3$. (B) $3x^2$.

(C) $15x^3$. (D) $15x^4$.

47. Consider the field $\mathbb{Q}(\sqrt{2}) := \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$. What is the inverse of the element $a + b\sqrt{2} \in \mathbb{Q}(\sqrt{2})$?

(A) $\frac{-a}{a^2+b^2} + \frac{b}{a^2+b^2}\sqrt{2}$.

(B) $\frac{-a}{a^2-2b^2} + \frac{b}{a^2+2b^2}\sqrt{2}$.

(C) $\frac{-a}{a^2+b^2} + \frac{b}{a^2-2b^2}\sqrt{2}$.

(D) $\frac{a}{a^2-2b^2} + \frac{b}{a^2-2b^2}\sqrt{2}$.

48. Which of the following matrix has the property $A^4 = A$?

(A) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

(B) $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

(C) $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$.

(D) $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}$.

49. Which one among the following is an example of a harmonic function ?

(A) $f(x, y) = x^2 + y^2$.

(B) $f(x, y) = \sin(x) + \sin(y)$.

(C) $f(x, y) = x^2 - y^2$.

(D) $f(x, y) = \cos(x) + \cos(y)$.

50. What is the remainder if $2\,2^{402}$ is divided by 11 ?

(A) 1.

(B) 4.

(C) 9.

(D) 7.