

## FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

## Mathematics

## MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours

Maximum : 80 Marks

## Part A (Objective Type)

*Answer all the twelve questions.**Each question carries 1 mark.*

1. If  $\alpha, \beta, \gamma$  are the roots of  $2x^3 + 3x^2 - x - 1 = 0$ . Find the equation whose roots are  $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$ .
2. State the Fundamental theorem of algebra.
3. Find a cubic equation, two of whose roots are given by  $1, 3 + 2i$ .
4. What do you mean by reciprocal equation. Give example.
5. What is the rank of the identity matrix of order  $n$ .
6. If  $A = [a_{i,j}]$  is an  $m \times n$  matrix and  $a_{i,j} = 7$  for all  $i, j$  then rank of A is \_\_\_\_\_.
7. A system of  $m$  homogeneous linear equations  $AX = 0$  in  $n$  unknowns has only trivial solution if \_\_\_\_\_.
8. For what values of  $a$  the system of equations  $ax + y = 1, x + 2y = 3, 2x + 3y = 5$  are consistent.
9. If the number of variables in a non homogeneous system  $AX = B$  is  $n$  then the system possesses a unique solution if \_\_\_\_\_.
10. Find the parametric equation of a line through  $P(3, -4, -1)$  and parallel to the vector  $i + j + k$ .
11. Find the unit tangent vector of the helix  $r(t) = \cos t i + \sin t j + t k$ .
12. Write equations relating rectangular and cylindrical coordinates.

(12 × 1 = 12 marks)

## Part B (Short Answer Type)

*Answer any nine questions.**Each question carries 2 marks.*

13. Solve  $4x^3 - 24x^2 + 23x + 18 = 0$ . Given that the roots are in arithmetic progression.
14. Transform  $x^3 - 6x^2 + 5x + 12 = 0$  into an equation lacking second term.

Turn over

15. If  $\alpha, \beta, \gamma$  are the roots of  $ax^3 + 3bx^2 + 3cx + d = 0$ . Find the value of  $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$ .
16. If  $A = \begin{pmatrix} -2 & -1 \\ 5 & 4 \end{pmatrix}$  Find  $A^{-1}$ .
17. Prove that the characteristic roots of Hermitian matrix are real.
18. If  $\alpha$  is an eigen value of a non singular matrix  $A$ , prove that  $\frac{|A|}{\alpha}$  is an eigen value of  $\text{adj } A$ .
19. Show that the product of characteristic roots of a square matrix of order  $n$  is equal to the determinant of the matrix.
20. Find the value of  $a$  for which  $r(A) = 3$  where  $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$
21. Find the velocity and acceleration vectors of  $r(t) = (3\cos t)i + (3\sin t)j + t^2k$ .
22. Find an equation for the circular cylinder  $4x^2 + 4y^2 = 9$  in cylindrical coordinates.
23. Evaluate  $\int_0^1 (t^3i + 7j + (t+1)k) dt$ .
24. Find the unit tangent vector for the circular motion  $r(t) = (\cos 2t)i + (\sin 2t)j$ .

(9 × 2 = 18 marks)

**Part C (Short Essay)**

*Answer any six questions.  
Each question carries 5 marks.*

25. If  $\alpha, \beta, \gamma$  are the roots of  $x^3 - x - 1 = 0$ . Find the equation whose roots are  $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$ .
26. Solve the equation  $x^2 - 12x - 65 = 0$  by cardan's method.
27. Solve  $x^3 + 6x^2 + 3x + 18 = 0$ .
28. Prove that the rank of a non singular matrix is equal to the rank of its reciprocal matrix.

29. Find the rank of  $\begin{pmatrix} 4 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ .

30. Using matrix method solve,

$$x + 2y + z = 2$$

$$3x + y - 2z = 1$$

$$4x - 3y - z = 3$$

$$2x + 4y + 2z = 4$$

31. Find the point in which the line  $x = 1 - t, y = 3t, z = 1 + t$  intersects the plane  $2x - y + 3z = 6$ .

32. Find the distance from the point S (0, 0, 1, 2) to the line  $x = 4t, y = -2t, z = 2t$ .

33. Find the eigen values and eigen vectors of  $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ .

(6 × 5 = 30 marks)

### Part D

*Answer any two questions.  
Each question carries 10 marks.*

34. Solve the equation  $6x^5 - 41x^4 + 97x^3 - 97x^2 + 41x - 6 = 0$ .

35. Verify Cayley Hamilton theorem for the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{pmatrix}$ .

36. Find the binormal vector and torsion for the space curve  $r(t) = (3\sin t)i + (3\cos t)j + 4tk$ .

(2 × 10 = 20 marks)