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C 4752 **(Pages : 3)** Name.......................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2021

(CBCSS)

Mathematics

MT 2C 10—OPERATIONS RESEARCH

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *the questions.*
Each question has weightage 1.

1. Verify whether $f(x) = x^2$ is a convex function or not.
2. Write the general form of a linear programming problem in two variables. Describe the method of solving such a problem using graphical method.
3. Prove that a vertex of the set of all feasible solutions S_F of a linear programming problem is a basic feasible solution.
4. Describe the concept of loop in a transportation array.
5. Define chain and path in graphs. Prove that path is a chain, but every chain is not a path.
6. Define cutting planes in integer programming.
7. State the minimax theorem in game theory.
8. What do we do in sensitivity analysis in linear programming problems ?

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **two** *questions from each unit.*
Each question has weightage 2.

UNIT I

9. Define the dual of a linear programming problem. Prove that dual of the dual is the primal problem.

10. Define multiplier vector and simplex multipliers. Explain their relevance in simplex method of solving linear programming problems.

11. Solve graphically the linear programming problem :

    Maximize $4x_1 + 2x_2$ subject to $x_1 + x_2 \leq 8, x_1 = 4, \; x_1 \geq 0, \; x_2 \geq 0$. Does the optimal solution change if the constraint $x_1 = 4$ is changed to $x_1 \geq 4$?

UNIT II

12. If the primal problem is feasible, prove that it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa.

13. Using dual simplex method

    maximise $2x_1 + 3x_2$ subject to $2x_1 + 3x_2 \leq 30, x_1 + 2x_2 \geq 10, x_1 \geq 0, x_2 \geq 0$.

14. Prove that the transportation problem has a triangular basis.

UNIT II

15. Describe the terms: chain, path, cycle, circuit, component and strongly connected with reference to graphs.

16. Describe the branch and bound method in integer programming.

17. Examine the payoff matrix $\begin{pmatrix} 1 & 3 \\ -2 & 10 \end{pmatrix}$ for saddle point.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Let $f(X)$ be a convex differentible function defined in a convex domain $K \subseteq E_n$. Then prove that $f(X_0), X_0 \in K$, is a global minimum if and only if $(X - X_0)' \nabla f(X_0) \geq 0$ for all $X \in K$.

(b) Use simplex method to verify that the problem : Maximize $f(X) = 2x_1 + x_2$ subject to the constraints $x_1 - x_2 - x_3 \leq 1,\ x_1 - 2x_2 + x_3 \leq 2,\ x_1 \geq 0, x_2 \geq 0,\ x_3 \geq 0$, has no finite optimal solution.

19. (a) Discuss the Caterer problem in operations research.

(b) Solve the transportation problem for minimum cost with the cost co-efficients, demands and supplies as given in the following table. Obtain three optimal solutions :

	D_1	D_2	D_3	D_4	
O_1	1	2	− 2	3	70
O_2	2	4	0	1	38
O_3	1	2	− 2	5	32
	40	28	30	42	

20. (a) Solve the following integer linear programming problem :

Maximize $\phi(X) = 3x_1 + 4x_2$; subject to $2x_1 + 4x_2 \leq 13, -2x_1 + x_2 \leq 2, 2x_1 + 2x_2 \geq 1,$

$6x_1 - 4x_2 \leq 15, x_1, x_2 \geq 0, x_1$ and x_2 are integers.

(b) By cutting plane method : Minimize $4x_1 + 5x_2$ subject to $3x_1 + x_2 \geq 2, x_1 + 4x_2 \geq 5,$ $3x_1 + 2x_2 \geq 7$; x_1, x_2 being non negative integers.

21. For an $m \times n$ matrix game, prove that both $\max_X \min_Y E(X, Y)$ and $\min_Y \max_X E(X, Y)$ exist and are equal.

(2 × 5 = 10 weightage)