C 61909 (Pages : 3) Name..............................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS—PG)

Mathematics

MT 4E 07—ADVANCED FUNCTIONAL ANALYSIS

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. Let X be a normed space and X' be the dual space. For $x \in X$, define the norm of x in terms of the elements of X'.

2. What is the dual of $\mathbb{K}^n$ with norm $\| \|_p$.

3. State the little moment problem in sequences.

4. If X is a reflexive normed space, prove that X' is also reflexive.

5. Define uniformly convex normed space. Interpret this concept geometrically.

6. Let X be a normed space and $A \in CL(X)$. Prove that every eigenspace of A corresponding to a nonzero eigenvalue of A is finite dimensional.

7. Define the complemented subspace property of a Hilbert space.

8. State the Unique Hahn-Banach extension theorem.

9. If $A \in BL(H)$ where H is a Hilbert space, then prove that $Z(A) = R(A^*)^{\perp}$.

10. If $A, B \in BL(H)$ are such that A and B are self-adjoint, then prove that A + B is also self adjoint.

11. Prove that the numerical range of an operator on a Hilbert space is a bounded subset of the field of scalars.

12. Let H be a Hilbert space over the field of complex numbers and $A \in BL(H)$. Prove that there are unique self-adjoint operators B and C on H such that $A = B + iC$.

Turn over

13. If A and B are positive operators, prove that A + B is also a positive operator.

14. Distinguish between eigenspectrum and approximate eigensPectrum of an operator on a Hilbert space.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Let X and Y be normed spaces. If $F \in BL(X, Y)$ and $G \in BL(Y, X)$, prove relation of transposes $(GF)' = F'G'$.

16. Let X and Y be normed spaces and $F \in BL(X, Y)$. Prove that

$$Z(F) = \{x \in X : x'(x) = 0 \text{ for all } x' \in R(F')\}$$

17. If x is the weak limit of (x_n) and y is the weak limit of (y_n), show that $x + y$ is the weak limit of $(x_n + y_n)$.

18. Let X be a Banach space which is uniformly convex in some equivalent norm. Prove that X is reflexive.

19. Let X be a normed space and $A \in CL(X)$. Prove that $\sigma_a(A) = \sigma(A)$.

20. Let H be a Hilbert space and $A \in BL(H)$. Prove that A is invertible if and only if its adjoint A* is inverible and in that case, $(A^*)^{-1} = (A^{-1})^*$.

21. Let H be a Hilbert space and $A \in BL(H)$. Prove that A is normal if and only if $\|A(x)\| = \|A^*(x)\|$ for all $x \in X$.

22. Let A be a compact operator on $H \neq 0$, prove that if A is self adjoint then $\|A\|$ or $-\|A\|$ is an eigen value of A.

23. Let H be a Hilbert space and $A \in BL(H)$. Prove that $k \in \sigma_a(A)$ if and only if $\bar{k} \in \sigma_a(A^*)$.

24. Let H be a Hilbert space and $A \in BL(H)$. Show that if A is compact, then A^* is also compact.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. State and prove Riesz representation theorem for L^p.

26. Let X be normed space. Then prove that X is reflexive if and only if every bounded sequence in X has a weak convergent subsequence.

27. Prove that a subset of a Hilbert space is weak if and only if it is bounded.

28. Let H be a Hilbert space and $A \in BL(H)$. Then prove that A or – A is a positive operator if and only if $|\langle A(x), y\rangle|^2 = |\langle A(x), x\rangle \langle A(y), y\rangle|$.

(2 × 4 = 8 weightage)